PRODROMUS

OF A

PRACTICAL TREATISE

ON THE

MATHEMATICAL ARTS:

CONTAINING DIRECTIONS FOR

SURVEYING AND ENGINEERING.

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PREFACE.

This book is chiefly made up of selections from a mass of heterogeneous materials, which I have been depositing in my common journal for more than thirty years—some of which, however, I published in 1830, under the title "Art without Science." I may add, that I had published a very small treatise under the same title, in the year 1800.

Students have made use of manuscript copies from my notes, for a kind of guide in their course of exercises, for the last four years. Examiners, appointed by the Patron of this Institute, have followed them, mostly, for the same period.

Though it is offered as the *Prodromus** *of a full treatise on Mathematical Arts*; I have progressed too far on the way to the *bourne of three score and ten*, to give any assurances. I have materials enough to complete the object; but "there is a point by nature fixed, whence life must downward tend."

Logarithms are not used in this book for purposes of calculation. It is a mistake to suppose that logarithms expedite calculations in trigonometry, *in common applications of it*. The tedious processes of multiplication and division, when we use natural sines, are overbalanced by the trouble of looking out logarithms and accommodating them to the various cases. This opinion is to be proved or disproved by trial alone. But in a long tedious process of several days labor, logarithms are generally useful.

Algebraic expressions are not used; because they are unnecessary, and very few are sufficiently versed in algebra to apply them to advantage. In truth, our lives are too short to devote much time to speculative mathematics. In past ages, when the science of nature was in its infancy, more time could be devoted to "mere tricks to stretch the human brain," than in this day of astonishing developements of nature's wonders.

* *Prodromos*, Greek, fore-runner.

Every teacher of experience knows, that the only successful mode of instruction is that which interests the student. Also that it is exceedingly difficult to excite interest by a blindfold course, whose object is not perceived. In learning land surveying, the student should always survey, under the mechanical direction of a teacher, before he studies the science of surveying. He should take latitude and longitude, and learn the use of the necessary instruments, before he devotes a day to the theory of lunar observations, &c. He will then perceive the object of his closet studies, and hear, understandingly and with delight, the lectures of his teacher. Even the common proportions of a triangle should not be introduced to a student's mind, until the teacher has directed him in the measurement of the lines and angles in true-earnest application. It is scarcely more absurd to attempt to theorize a blacksmith's boy into horse-shoeing, than to attempt to make a practical mathematician without out-of-doors practice.

But the extreme of absurdity is most emphatically exhibited by putting books into students' hands, written in a language which they cannot understand. If they must read a language which is new to them, they must have time to learn it. An honest teacher ought, in such cases, to make the students or their guardians, understand that they are not to study mathematics, until they have devoted a year to the study of a new language, called algebra.

In this small work, an attempt is made to enable a common-sense farmer, mechanic, merchant, or other man of business, who is but an ordinary arithmetician, to become sufficiently qualified for the business concerns of life, as a practical mathematician. But he must be shewn the use of instruments; as it is an idle waste of time to attempt to learn their use from books.

AMOS EATON.

Rensselaer Institute,
Troy, March, 1838.

POCKET SCALE

Of Natural Sines, Chord Line, and Equal Parts.

A six inch pocket ruler has always been considered as essential to a mechanic. To a mathematician, or even an ordinary traveller, &c., such a measure should furnish the necessary *scales* of what we need most. A scale of equal parts of an inch in tenths, and a diagonal inch for hundredths, are the most important. A line of chords, of about a three inch sweep of sixty, is generally deemed next in importance. Geometrical trigonometry may be wrought by these two scales. But our situation is not always (nor even at one occasion in one hundred) such, that we can sit down at a table, and use the scale and dividers accurately. But we can make calculations with natural sines, while riding in a stage, or sitting at the theatre, or a concert. But this operation demands a table of *natural sines.* A book, then, of many pages, must be carried in our pockets. To obviate this difficulty, I have prepared the annexed *Abridged Table of Natural Sines.* Particular directions for its use are hereunto subjoined. A six inch ruler, 2 inches wide, will soon be made in Troy, N. Y., containing the diagonal scale of equal parts, the chord line, and the abridged table of natural sines, carried out to sines of degrees and minutes. It will always be found at bookstores, where this Prodromus is sold, after the artist has completed his instruments for making the ruler.

ABRIDGED TABLE OF NATURAL SINES.

At page 31, a table with this heading is printed. It was hurried into that form for cases where perfect exactness in the minutes should not be required. It was afterwards discovered, that by extending the *augments, degrees and minutes* might always be calculated and used, when books, containing full tables, were not at hand.

DIRECTIONS

For using the Abridged Table of Natural Sines, here inserted—also, on the Pocket Ruler.

1. If the sine of a *whole degree*, or of a *half degree* (30 minutes) is required, it is found against such degree or half degree (that is, such degree and 30 minutes.)

2. If the sine of any number of minutes, less than 30 degrees, is required, proceed as follows: Find the number in the column of augments, against the last degree preceding the number of minutes, whose sine is required. Multiply this amount by the number of minutes, and add the product to the sine of the said preceding degree; setting the right hand figure one place to the right of the sine. The sum will be the sine of the degrees and minutes required.

3. If the sine of any number of minutes more than 30, is required, proceed thus: Find the number in the column of augments, against the last 30 minutes preceding the minutes whose sine is required. Multiply this augment by the number of minutes exceeding said 30 minutes; and add the product to the said sine of said preceding 30 minutes—setting the right hand figure one place to the right of the sine. The sum is the sine of the degrees and minutes required.

4. In an operation in trigonometry (where natural sines are used, see sec. 39) if the answer is in sines, find the degrees and minutes as follows: Find in the table the nearest degree or half degree, next less than is required. Subtract that sine from the said answer, and divide the remainder by the augment set against the degree or half degree; which will give the additional minutes. These added to said nearest degrees and minutes, give the true degrees and minutes required.

5. In most cases of ordinary practice, the subdivisions of a degree into six parts (10 minutes each) will be sufficiently accurate. Every sixth division (that is, every 10 minutes) requires a very simple application of the augments. Ten and 40 minutes require the augment once merely, without extending a figure to the right of the sine—20 and 50 minutes require the augment doubled, and not extended to the right of the sine.

ABRIDGED TABLE OF NATURAL SINES.

Aug-ments.	De-grees.	S. C-S.	C-S. S.	De-grees.	Aug-ments.	Aug-ments.	De-grees.	S. C-S.	C-S. S.	De-grees.	Aug-ments.
291	0.00	0.00000	1.00000	90.00		267	23.00	.39072	.92050	67.00	112
291	.30	.00873	.99996	.30	1	266	.30	.39875	.91706	.30	114
291	1.00	.01745	.99985	89.00	3	265	24.00	.40674	.91355	66.00	117
291	.30	.02618	.99966	.30	6	264	.30	.41469	.90996	.30	120
291	2.00	.03490	.99939	88.00	8	263	25.00	.42262	.90631	65.00	121
290	.30	.04362	.99905	.30	11	262	.30	.43051	.90259	.30	123
290	3.00	.05234	.99863	87.00	13	260	26.00	.43837	.89879	64.00	126
290	.30	.06105	.99813	.30	16	259	.30	.44620	.89493	.30	128
290	4.00	.06976	.99756	86.00	18	257	27.00	.45399	.89101	63.00	130
290	.30	.07846	.99692	.30	21	556	.30	.46175	.88701	.30	133
290	5.00	.08716	.99619	85.00	24	256	28.00	.46947	.88297	62.00	135
290	.30	.09585	.99540	.30	26	255	.30	.47716	.87882	.30	137
290	6.00	.10453	.99452	84.00	28	253	29.00	.48481	.87462	61.00	140
289	.30	.11320	.99357	.30	31	252	.30	.49242	.87036	.30	141
288	7.00	.12187	.99255	83.00	34	251	30.00	.50000	.86603	60.00	143
288	.30	.13053	.99144	.30	37	250	.30	.50754	.86163	.30	145
287	8.00	.13917	.99027	82.00	39	248	31.00	.51504	.85717	59.00	147
287	.30	.14781	.98902	.30	41	246	.30	.52250	.85264	.30	151
287	9.00	.15643	.98769	81.00	43	245	32.00	.52992	.84895	58.00	153
286	.30	.16505	.98629	.30	46	244	.30	.53730	.84339	.30	155
286	10.00	.17365	.98481	80.00	49	243	33.00	.54464	.83867	57.00	157
286	.30	.18224	.98325	.30	52	242	.30	.55194	.83389	.30	160
285	11.00	.19081	.98163	79.00	54	240	34.00	.55919	.82904	56.00	162
284	.30	.19937	.97992	.30	57	238	.30	.56641	.82413	.30	163
283	12.00	.20791	.97815	78.00	59	237	35.00	.57358	.81915	55.00	165
283	.30	.21644	.97630	.30	62	236	.30	.58070	.81412	.30	167
283	13.00	.22495	.97437	77.00	64	234	36.00	.58779	.80902	54.00	170
282	.30	.23345	.97237	.30	66	232	.30	.59482	.80386	.30	172
282	14.00	.24192	.97030	76.00	69	230	37.00	.60182	.79864	53.00	174
281	.30	.25038	.96815	.30	72	228	.30	.60876	.79335	.30	176
280	15.00	.25882	.96593	75.00	74	227	38.00	.61566	.78801	52.00	178
280	.30	.26724	.96363	.30	77	226	.30	.62251	.78261	.30	180
279	16.00	.27564	.96126	74.00	79	225	39.00	.62932	.77715	51.00	182
278	.30	.28402	.95882	.30	81	223	.30	.63608	.77162	.30	184
277	17.00	.29237	.95630	73.00	83	222	40.00	.64279	.76604	50.00	186
277	.30	.30071	.95372	.30	86	220	.30	.64945	.76041	.30	187
276	18.00	.30902	.95106	72.00	88	218	41.00	.65606	.75471	49.00	189
275	.30	.31730	.94832	.30	91	216	.30	.66262	.74896	.30	191
274	19.00	.32557	.94552	71.00	93	215	42.00	.66913	.74314	48.00	193
274	.30	.33381	.94264	.30	96	213	.30	.67559	.73728	.30	195
273	20.00	.34202	.93969	70.00	98	211	43.00	.68200	.73135	47.00	197
272	.30	.35021	.93667	.30	101	209	.30	.68835	.72537	.30	199
271	21.00	.35837	.93358	69.00	103	207	44.00	.69466	.71934	46.00	201
270	.30	.36650	.93042	.30	105	206	.30	.70091	.71325	.30	203
269	22.00	.37461	.92718	68.00	107	205	45.00	.70711	.70711	45.00	205
268	.30	.38268	.92388	.30	110		.30	.71325	.70091	.30	

☞ Students are not to apply the above Table until they have studied sections 31 to 40, inclusive.

CORRECTIONS TO BE MADE WITH THE PEN.

Page 13, top line—"seventeen" change to "four."

Page 31—the augments are better on page vii.

Page 80, sec. 141—"foot" change to "inch."

Page 91—change places of the words "Top" and "bottom."

Page 100, 3d line—after "Station" read "No. 10."

Page 100, 3d line—instead of "moved up his instrument" read "went with the Targetman."

Page 100, 19th line—for "Sec. 000" read "Sec. 227."

Page 100, 23d line—for "740" read "760."

Page 103, 1st line—for "Station" read "two stations."

Page 111, last of sec. 216—between "the" and "ordinate" interline "square of."

Page 114, sec. 224—"chair" change to "bench."

Page 122, sec. 253, middle line—strike out "the square of."

Page 133, sec. 288, near the end—"to the root add" change to "from the root subtract."

REFERENCES.

THE student is referred to last part of the book, where wood-cut figures are described, for continuations of several sections of importance.

First is from section 198 to 200, extending and illustrating by wood-cut figures, the method of *calculating rail-road curves.*

Second is from sections 211 to 214, extending and illustrating by wood-cuts, the method of calculating *ordinates*, for offsets from secondary chord lines, one hundred feet each.

Third is from sections 224 to 226, extending and illustrating by wood-cuts, the method of *calculating excavations* and *embankments.*

This *text-book* is not limited in its object to students in *surveying* and *engineering.* Not more than 43 pages are, exclusively, devoted to them. In it will be found those rules and directions for calculations, which are essential to every correct student in Geography, Astronomy, and Natural Philosophy; also to all classes of readers, who wish to understand what they read.

Teachers of female institutions, and of common academies, are requested to look over the contents, and consider the manner in which subjects are treated.

Such institutions may omit—

1. PRACTICAL LAND SURVEYING, from page 47 to page 74.
2. RUNNING OUT RAIL-ROADS, from page 95 to page 111.

All this treatise, excepting the above excepted 43 pages, should be studied and illustrated with practice by every student, who is presented to the public as tolerably educated.

The common practice of introducing a system of elementary rules and demonstrations, so to cumber a small practical treatise as almost to exclude the professed object, has always appeared to be absurd. Elementary treatises, executed in a style of excellence which cannot be surpassed, are to be found in almost every book-store. To them the learner is referred; and will not be taxed with the reprint, and expense of copy-right, for the sake of swelling a small work into a large one. Gibson's System of Surveying, for example, a work of almost 500 pages 8vo, contains less than 100 pages, which are devoted to the professed object of the work. It is hoped, that the learner will find nothing in this, which is useless in aid of his proposed object.

With pleasure I acknowledge my obligation to State engineer Holmes Hutchinson, Esq., for his instructive explanations in answer to my numerous

inquiries for the last half dozen years. To engineer Wm. C. Young I am also indebted for much useful information on the construction of rail-road works; as exemplified and explained by him at the extensive works in Schenectady. For the latest and most approved method of *rail-road surveying*, including staking out, running curves, measuring for excavations, &c., students are referred to articles furnished by engineers Sargent and Evans. I will take this opportunity to say, that without exceptions, every practising engineer with whom I have had any intercourse during the existence of the Engineer department at this Institute (four years) has manifested a strong desire to aid its progress and extend its influence.

CONTENTS.

THIS table of contents is constructed for the convenience of examiners. As the by-laws of this Institute forbid all persons concerned in the instruction of students giving any opinion on the subject of their qualification, and as each board of examiners is made up of gentlemen who are wholly disconnected with the school; it was supposed, that few would be willing to devote sufficient time to each subject to extract the essential points in it. Therefore each item of the contents is made to contain what appears to be sufficient to give the student a fair clue to all that is expected from him.

N. B. The word *Explain*, is supposed to be prefixed to every subject, in the *imperative mode*. In numbering sections, *inclusive* is understood.

ARITHMETIC.

TRIGONOMETRY.

MENSURATION.

LAND SURVEYING.

EXCAVATIONS AND EMBANKMENTS.

CANALS.

ROADS IN GENERAL.

WATERWORKS.

USEFUL ROCKS AND CEMENTS.

TIMBER MATERIALS.

IRON MATERIALS.

MATHEMATICAL ILLUSTRATIONS.

Before studying this treatise, students must have been sufficiently exercised under teachers' dictum, with whole numbers and decimals, in Addition, Subtraction, Multiplication and Division, and the Rule of Three. Nothing more is required; neither is it profitable to detain students with Compound Arithmetic, Vulgar Fractions, or Algebra, until they are made acquainted with the most useful parts of the Mathematical Arts—particularly the general applications of superfices, solids, and trigonometry, to common business concerns. This I assert; and my assertion is founded on forty years' experience. And this rule applies to merchants' clerks and others, whose operations are merely arithmetical. The power of numbers, once understood, applies to all cases alike. Therefore the mere land surveyor makes a better book-keeper by one month's practice, than the student in mere book-keeping does in a year. The reason is manifest. The surveyor takes, necessarily, a scientific view of the power of number; while the student in book-keeping takes a parrot-like rotine of artificial forms. The former is governed by sound reason—the latter is led blind-fold by authority. He is a mere machine—but the mathematician is disciplined as an intellectual being.

Alcohol and Algebra are Arabic names. *Alcohol* is a powerful agent, of vast importance. But its abuses render it a curse. *Algebra* is a powerful entering wedge in Mechanics, and of great importance in the concise expressions of valuable formulæ. But a kind of affectation of technical learning has so far obscured the mathematical arts with algebra, as to render it an absolute nuisance. In this little treatise, algebraic formulæ are translated into fair English. Biot, (a most distinguished French Philosopher,) after thirty years' experience, as teacher in the algebraic mode of expression, prepared a System of Natural Philosophy, totally divested of all such technical obscurities. His authority, supported by success, has, in a great measure, revolutionized the course of mathematical learning in France.

ELEMENTARY OPERATIONS WITH NUMBERS.

Sec. 1. The science of numbers, called Arithmetic or Mathematics, may be resolved into three elementary operations: *Addition*, *Separation* and *Notation.*

Addition.

Sec. 2. This operation consists in uniting individuals into groups, masses, or sums; as the arranging of 100 soldiers into a group, called a company—uniting 196 pounds of flour in a mass, called a barrel—uniting the value of 100 cents into a silver coin, called a dollar—or uniting in one sum a sufficient number of feet of plank for laying a floor in a room of given length and breadth.

Sec. 3. When several additions are performed by one operation, we distinguish this modification of addition by the descriptive name, *Multiplication:* as $12 paid to each of 7 laborers must be added seven times to find out the whole sum to be paid, thus: $12+12+12+12+12+12+12=84$. But we may learn by rote to add thus: seven times twelve equals 84. This *rotine method* of adding is called *Multiplying.*

Separation.

Sec. 4. This operation consists in taking part from the whole; or separating smaller portions of masses, or numbers, from the larger. Individuals of groups are separated from each other, or distributed into smaller groups. The operation is called *Subtraction*, or *Division*, according to its peculiar application. All results may be produced by Subtraction; but Division is more expeditious when a separation into numerous parcels or parts is required, and particularly when the proportional parts are to be ascertained, from given data, for proportional separation.

If 1281 dollars are to be equally divided among 61 laborers, we divide the dollars by 61, by a tabular rotine, called Division, thus:

61)1281(21—giving $21 to each.
122
———
61
61
—
00

Sec. 5. The same result will be produced by perpetually subtracting 61 from 1281, and counting up the number of subtractions, thus:

$1281		854		427	
61	1st.	61	8th.	61	15th.
1220		793		366	
61	2d.	61	9th.	61	16th.
1159		732		305	
61	3d.	61	10th.	61	17th.
1098		671		244	
61	4th.	61	11th.	61	18th.
1037		610		183	
61	5th.	61	12th.	61	19th.
976		549		122	
61	6th.	61	13th.	61	20th
915		488		61	
61	7th.	61	14th.	61	21st.
854	carried up.	427	carried up.	00	

As 61 can be subtracted 21 times from $1281, each of the 61 laborers will have $21, as given by dividing by 61.

Notation.

Sec. 6. *The operation of setting down or recording numbers.* Numbers are expressed by Roman letters, or by Arabic figures. But Roman letters are never used in the process of calculations. They are very convenient for expressing the numbers of large divisions which are to be subdivided: such as Classes of Plants, expressed in Roman letters, which are subdivided into *Orders*, and expressed in figures.

Sec. 7. The Roman letters used for expressing numbers are, I for one, V for five, X for ten, L for fifty, C for a hundred, D for five hundred, M for a thousand. When these letters are joined in a horizontal row from left to right, with the smallest valued letter at

the right, that is added to the larger. But if the smaller valued letter is set on the left of the larger, it is subtracted. Thus, VI stands for six, and IV for four; XI for eleven, IX for nine; LX for sixty, XL for forty; CX for one hundred and ten, XC for ninety. No letter, however, but I, X and C, is used by us in this manner as a subtrahend.

Sec. 8. The figures are, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. When joined in a horizontal row, they begin their value with the right hand figure; and the figure, added on its left, stands for ten times its value when alone: the next for one hundred times as much: the next for one thousand times, and so on. In the notation of figures they are pointed off in threes, from right to left.

Sec. 9. The *element* of the *first* three numbers is *unit*—of the *second* three, is *thousand*—of the *third* three, is *million*—of the *fourth* three, is *billion*—of the *fifth* three, is *trillion*—of the *sixth* three, is *quadrillion*—of the *seventh* three, is *quintillion*—of the *eighth* three, is *sextillion*—of the *ninth* three, is *septillion*—of the *tenth* three, is *octillion*—of the *eleventh* three, is *nonillion*—of the *twelfth* three, is *decillion*—of the *thirteenth* three, is *undecillion;* and so on, indefinitely, following the Latin numerals, with the same Anglicising termination.

Quindecillions.	hundreds of—	tens of—	Quadridecillions.	hundreds of—	tens of—	Tredecillions.	hundreds of—	tens of—	Duodecillions.	hundreds of—	tens of—	Undecillions.	hundreds of—	tens of—	Decillions.	hundreds of—	tens of—	Nonillions.	hundreds of—	tens of—	Octillions.	hundreds of—	tens of—	Septillions.
17			16			15			14			13			12			11			10			9
4,	8	6	7,	4	3	9,	2	4	5,	6	1	7,	6	3	5,	9	2	6,	7	3	5,	7	2	4,

hundreds of—	tens of—	Sextillions.	hundreds of—	tens of—	Quintillions.	hundreds of—	tens of—	Quadrillions.	hundreds of—	tens of—	Trillions.	hundreds of—	tens of—	Billions.	hundreds of—	tens of—	Millions.	hundreds of—	tens of—	Thousands.	hundreds of—	tens of—	Units.
		8			7			6			5			4			3			2			1
7	8	9,	6	3	5,	2	4	6,	8	5	7,	8	4	6,	4	5	6,	7	8	9,	5	6	7

Sec. 10. This example is to be read as follows: seventeen *quindecillions*, eight hundred and sixty-seven *quadridecillions*, four hundred and thirty-nine *tredecillions*, two hundred forty-five *duodecillions*, six hundred seventeen *undecillions*, six hundred thirty-five *decillions*, nine hundred twenty-six *nonillions*, seven hundred thirty-five *octillions*, seven hundred twenty-four *septillions*, seven hundred eighty-nine *sextillions*, six hundred thirty-five *quintillions*, two hundred forty-six *quadrillions*, eight hundred fifty-seven *trillions*, eight hundred forty-six *billions*, four hundred fifty-six *millions*, seven hundred eighty-nine *thousands*, five hundred fifty-seven.

Sec. 11. Gregory says, this is the method of notation adopted in France, Germany, &c., which he prefers. But the English often proceed by repeating *millions;* as millions of millions of millions of millions, &c. Take the following example of using *million* as a substitute for *billion, trillion,* &c.:

6 5 4 3 2 1
8,654,327,923,578,263,523,127,354,687,128,354,627.

8 millions of millions of millions of millions of millions of millions, 654,327 millions of millions of millions of millions of millions, 923,-578 millions of millions of millions of millions, 263,523 millions of millions of millions, 127,354 millions of millions, 687,128 millions, 354,627.

Sec. 12. The *language of number* is divided into *Cardinal* and *Ordinal* adjectives. As one, two, three, four, five, six, &c., are *cardinal* names. First, second, third, fourth, fifth, sixth, &c., are *ordinal* names. Ordinal names, or adjectives, are changed to adverbs by adding *ly*—as fifthly, tenthly, twenty-eightly, &c.

Common Characters.

Sec. 13. A point, or period, as a character, is important in notation. It should always stand at the right of a horizontal series of integers, if decimals are appended. Thus: 62754 inches, and 726 thousandth of an inch, is thus expressed, 62754.726. In enumerating we begin at point, and say, units, tens, hundreds, thousands, tens of thousands, on the left—and tens, hundreds, thousands, on the right. We therefore read this example thus: sixty-two thousand, seven hundred and fifty-four inches; *point.* seven, two, six.

Sec. 14. *Double points,* or colons, are used in the Rule of Three, thus: if \$70 buy 9 acres, what will \$926 buy? Is set down, \$70 : 9 a. :: \$926. The first single colon is to be read "*is to;*" the last is read "*to;*" and the double colon is to be read "*so is.*" Therefore the true reading of the above example is, as \$70 *is to* 9 acres, *so is* \$926 *to* 119 $\frac{4}{70}$ acres—same as 70 : 9 :: 926 : 119 $\frac{4}{70}$.

Right Cross +, (*called plus,*) signifies that the number following it is to be added to something preceding, as 267+31, shews that 31 is to be added to 267, making 298.

Oblique Cross ×, signifies that the number following it, is to be multiplied into something preceding, as 124×12, shews that 12 is to be multiplied into 124, making 1488.

Horizontal line —, (*called minus,*) signifies that the number following it is to be subtracted from something preceding, as 729—36 shows that 36 is to be subtracted, leaving 693.

Horizontal dotted line ÷, signifies that the number following it, is to be used as a divisor for dividing some dividend that precedes it, as 806÷26, shews that 26 is to be used as a divisor for the dividend 806, making 31. This dotted line is often omitted, and the figures set down like an improper vulgar fraction, thus, $\frac{806}{31}$.

Parallel lines =, signify that the number or numbers following them, equal something preceding, as 17×5×4=340.

Inverted figure seven √, signifies that what follows it, requires the square root to be extracted. Figure 3 over it implies that the cube root should be extracted, &c., as $\sqrt[3]{27}$ requires the cube root to be extracted, making 3.

N. B. Students must be exercised in notation, until they are familiar with the right application of the characters, and arrangement of figures.

Decimals.

Sec. 15. Decimals are parts of integers (whole numbers,) pointed off to the right by a period. The first figure expresses tenths of an integer, the second hundredths, and so on, diminishing ten fold at every figure. Several figures of decimals express a general

proportion of an integer, as 9.6345 inches expresses nine inches and 6345 ten thousanths of an inch. But the most common as well as most convenient mode of expression is, to read off the figures separately after the point—thus: nine, point. six, three, four, five. In reading off Natural Sines, or Logarithms, this method is always to be adopted.

Addition of Decimals.

Sec. 16. Decimals, or integers and decimals together, are always added like whole numbers. But in setting them down for adding, points must be set under each other in a column; and let the figures on both sides of each point stand at uniform distances. This will cause some lines to project further to the right and left than others. Of course care is required in footing up the figures in their proper columns.

Examples in Addition of Decimals.

76543.6201	63421.6203
360.2	9.
2.76423	243.67861
4239.621	27.
7.3	3.00001
36012.00034	789.9
117165.50567	64493.19892

Subtraction of Decimals.

Sec. 17. Decimals, or integers and decimals together, are always subtracted like whole numbers. But in setting them down for adding, the point in the subtrahend must be set directly under the point in the minuend; and the figures each side of the point stand as directed in addition.

Examples in Subtraction of Decimals.

63967.9342	80001.
201.7	2.0096
63766.2342	79998.9904

Multiplication of Decimals.

Sec. 18. Decimals, or decimals and integers together, are always multiplied like whole numbers. But after multiplying, care is required in placing the point between the integers and decimals. The rule is, to point off for decimals just as many places of figures, as are pointed off in both the multiplier and multiplicand; leaving for integers all the rest, if any.

Examples of Multiplication of Decimals.

Diameter of a circle, 643.231 inches. Formulæ for circumference, 3.1416 inches.

643.231	(operation
3.1416	omitted.)
2020.7745096	

Four decimal places in the multiplier, and three in the multiplicand, require seven to be pointed off. Therefore the circumference is 2020 inches and 77 hundredths, if hundredths come near enough. But if exactness is required, say 2020 inches, and point .7745096 decimals of an inch.

Area of a square embracing a circle is	5402064.201
Formula for reducing a square to a circle,	.7854
	4242781.2234654

Four decimal places in the multiplier, and three in the multiplicand, require seven to be pointed off. Therefore the area of the circle is as above—that is, 4242781 square inches, and 22 hundredths, if hundredths come near enough. Or, point .2234654 decimals of an inch.

Division of Decimals.

Sec. 19. Decimals, or decimals and integers together, are always divided like whole numbers. But, after dividing, care is required in placing the point between the integers and decimals. The rule is, to point off for decimals in the quotient so many places of figures, as to make the number pointed off *in the divisor and quo-*

tient, just equal the number pointed off *in the dividend alone.* But if there are not as many pointed off in the dividend as in the divisor, cyphers must be added to the dividend (beyond the point) to make the number equal. And if an equal number does not carry on the decimals of the answer far enough for the required exactness, more may be annexed—always applying the rule for pointing off, as before stated.

N. B. We often proceed by adding decimals, in carrying on the decimals in the quotient to greater extent than those of the dividend will admit. In such cases, all the added decimals must be counted as if they had previously been added to the dividend.

Examples in Division of Decimals.

The area of a circle is 4242781.2234654 inches; divide by the formula .7854, which will give the area of a circumscribing square:

.7854)4242781.2234654((operation
5402064.201 omitted.)

Here are four places pointed off in the divisor and seven in the dividend. Now there must be pointed off in the quotient, which added to those of the divisor (being all the divisor) equal those of the dividend. The answer then is, that the circumscribing square made about the given circle, contains 5402064 square inches, and point .201 decimals of a square inch.

Sec. 20. *Bring compound expressions to Decimals.* In all cases where measures, weights, or values of any kind, are required to be brought into decimal expressions, make a vulgar fraction expressing the proportions. Then divide the numerator by the denominator; adding cyphers to the numerator as far as may be required.

Bring £16 7s. and 4d. to the decimal of a pound: 7s. and 4d.= 88d. A penny is the 240th of a pound. Then seven and fourpence is $\frac{88}{240}$ of a pound—240)88.0000(.3666. Answer is £16. 3666. Note: this results in a circulating decimal; as it will forever give 6, and is ever approximating the truth.

Rule of Three.

Sec. 21. When a value is set upon one article, it is self-evident that the value of any number of articles may be found, by multiply-

ing the value of one by the number whose whole value is required. As, if one square foot of ground costs 60 cents, and a house lot may be had at the same price per foot, which contains 1800 feet, it is manifest that the whole lot will cost 1800 times 60 cents, or 1080 dollars.

This stated, according to the form of the Rule of Three, will stand thus:

1 : \$0.60 :: 1800 : \$1080.00.

Here the second and third numbers are to be multiplied together to produce the fourth for the answer. But if the first number is more than one, the fourth number must be divided by it; or it will be too great.

Therefore, if four feet should cost but 60 cents, then the answer would be four times too much, and must be divided by 4—thus:

```
4 : $0.60 :: 1800 : $270.00
                 .60
            --------
           4)1080.00
              270.00
```

Therefore, these arrangements and operations meet all possible cases, where we can say—if the first number gives, produces, purchases, &c., the second, what will the third give, produce, purchase, &c. The second and third being inter-multiplied, always give the answer, on the supposition that the first number contains but one of the articles under calculation. If it contains more than one, the answer must be reduced to the truth, by dividing by it.

Example. If the excavation of 27 yards of earth cost \$17.25, what will the excavation of 77 yards cost?

```
27 : 17.25 :: 77 : 49.19
           77
   ----------
   )1328.25(49.19
```

If one yard of excavation should cost the \$17.25, the 1328.25 would be the answer without being divided by 27. But as the whole 27 yards cost but \$17.25, that answer would be 27 times too much, if not reduced by this divisor.

ROOTS AND POWERS.

Any number is called a root, when considered in relation to its powers; and its powers are estimated by the times it is multiplied by itself. As $2\times2=4$ is 2d power—$2\times2\times2=8$ is 3d power—$2\times2\times2\times2=16$ is 4th power—$2\times2\times2\times2\times2=32$, 5th power, &c.

There cannot be a real exhibition of any power above the 3d, or cube. But it is often necessary in a calculation to advance, ideally, into the higher powers.

No student is qualified for entering upon the study of the Mathematical Arts, without a good knowledge of the square root and of the cube root. These are made as familiar, in this treatise, as any diligent student can desire.

Hutton's concise method for approximation in all the higher powers, (sufficient for ordinary practice,) is given at the end of the roots.

SQUARE ROOT.

Sec. 22. A distinction between the *Squáre* and the *Root* must be understood by students. A root multiplied by itself produces the square: as the *root* 4 multiplied by itself, produces the *square* 16. When we use the word *root*, with or without the word *square* prefixed, we mean the *root only*. When we intend to express what is produced by multiplying the root by itself, we use the word *square only*.

Operations under the head of square root are divided into *Involution* and *Evolution*. It is an operation in Involution to multiply 10 by 10 and produce 100. It is an operation in Evolution, having 100 given, to find the number which multiplied by itself will produce the given hundred. The operation is called extracting the root.

Involution of the Square Root.

Sec. 23. This process requires no instruction. It is simple multiplication, limited to multiplying the same number by itself; as twelve times twelve give one hundred forty-four.

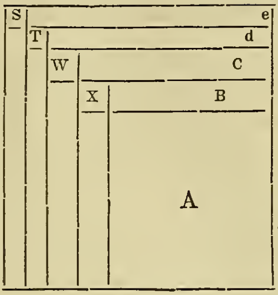
Evolution of the Square Root.

Sec. 24. This operation, when applied to squares, whose roots are found in whole numbers, within the limits of common multiplication tables, is a mere mental operation requiring no figures. As the root of 144 is 12—of 121 is 11—of 100 is 10—of 81 is 9—of 64 is 8—of 49 is 7—of 36 is 6—of 25 is 5—of 16 is 4—of 9 is 3—of 4 is 2—of 1 is 1. No intermediate case is found among these examples, which can be thus easily and mentally solved. But all the intermediate squares will give fractions of numbers in their roots. And all higher numbers present analagous difficulties, and still more complicated. Hence Evolution, or the Extraction of the Square Root, requires study and method.

Sec. 25. *Directions for extracting the Square Root.* As the number of figures in the root will always be equal to the number of pairs in the square of it, the pairs are pointed off from right to left. If an odd figure remains at the left end of the line, this alone will give one point, and of course stands in the place of a pair. Therefore we can foresee the number of figures which the root will contain, as soon as the square is pointed off. The last pair, or point, on the left, gives a number, which always contains more than half of the whole root; consequently that point in the square exceeds, in breadth, all the rest of the area of the square.

Sec. 26. Take for example 45623.0000 square chains. These chains we wish to lay out in a square piece of ground. One side of this square lot will be the root of these square chains. There-

fore it is a side of the lot we are in search of. As the square of the first point at the left will be 2, and this must have two figures at the right of it, besides the decimals, the nucleus square A must be 200 chains on each side. The slip B, 10—slip C, 3—slip d, .5—slip e, .09.



e————.09
d————.5
C————3.
B————10.

A————200.

213.59 is the length of one side of the lot, or the root of the given square.

Operation.

```
          45623.0000(213.59
          4
          —
X      41)56
          41
          ——
W     423)1523
          1269
          ————
T    4265)25400
          21325
          —————
S   42709)407500
          384381
          ——————
           23119+
```

N. B. The teacher must shew the student to perform similar operations frequently; then he will be prepared to understand this illustration.

CUBE ROOT.

Sec. 27. The same distinction between *Involution* and *Evolution* must be understood by students under the cube root as that described under the square root.

Evolution of the Cube Root.

Sec. 28. As the number of figures in the cube root will always be equal to the number of tripple figures in the sum given for extraction, the given figures are pointed off in threes from right to left. If two or one remain, a separate point is also made of such remainder. Therefore we can foresee the number of figures, which the root will contain, as soon as the given figures are pointed off, as mentioned under the square root. The cube root of the last point on the left will exceed that of all the remaining points; as all the rest must stand on its right, and, of course, diminish tenfold in value. (Cyphers, equalling the number of points, may be annexed to it.) This cube root will be the length (or linear extent) of the nucleus cube; upon three sides of which all that remains to be extracted is placed in layers. Therefore the operation is to be so performed, as to give the thicknesses and superfices of all outer layers, and of the filling-in corners.

Directions for Extracting the Cube Root.

Operation.

```
Cube    75686967(423   Root.
        64
        -----
   5044)11686
        10088
        -----
 532989)1598967
         1598967
```

Auxiliary operation carried on with the above.

1st. $4\times4\times4=64$ square of the nucleus cube.

2d. $4\times4=16$ area of each face of the nucleus cube. $16\times3=48$ area of the three faces, being the trial divisor.

```
48)116(2  thickness [last figures left off, 86.]
   96
   --
   20
```

4+4+4=12 length of the three parallelopipeds.
Length of the cube —2 set to the right, as of less value.

	122	
Width same	2	
	244	area of all the cube parallelopipeds.
Area of the faces	48	
	5044	true divisor for second figure.

3d. $42\times42=1764\times3=5292$ area of the three faces.

5292)15989(3 [off 57] $42+42+42=126$ length of parallelo-
thickness 3 [pipeds.

1263
3

So on like the rest above, &c.

Sec. 29. *Explanation.* Students must inspect a model, while examining this calculation. If no wooden model is prepared, proceed thus: Cut a potatoe cube, about one inch. Cut three potatoe slabs one fourth of an inch thick, whose area (each) will precisely cover three of the sides of the cube—pin them on their respective sides, so that when on, the whole shall be an enlarged cube; but with unfilled corners. Fill the three long corners with potatoe parallelopipeds; and fill the place where all the pieces leave an open corner, with a cube of the same thickness of the parallelopipeds.

By the operation annexed the nucleus cube is obtained by trial; that is, by multiplying the nearest figures, until the nearest is found within the point (75 in this case,) as $5\times5\times5=125$—this being too high, say $4\times4\times4=64$—this being the nearest below 75, consider 4 the measure of the nucleus cube.

After bringing down the next point, divide by a trial divisor, made up of the three areas to be covered with slabs; the quotient will be their thickness. The same thickness will equal the breadths of the parallelopipeds and of the corner cube. Their united length multiplied by their breadths, and added to the aforesaid areas, give the whole area.

Explanations. This area is the true divisor of the 11686 in this example. In adding the area of the three faces, of the parallelopipeds, and of the corner cube, their respective areas must be arranged from left to right according to their values. In this case, 48 is, truly, 4800—12 is 120—2 is unity. Therefore they add thus:

$$\begin{array}{r} 2 \\ 120 \\ 4800 \\ \hline 4922 \end{array}$$

But the areas of the parallelopipeds and corner cubes being obtained by a joint multiplication, they form 244 before 48 is added. This explains that position of them.

Students will perceive that the area of but one face of the parallelopipeds and corner cube are calculated. This is on account of their mitred character; as the middle of each side, only, is reckoned in their areas. (See figure.) The lengths of the two dotted lines give the area, which is equal to the whole of one side; therefore one side, only, is taken.

Sec. 30. *Hutton's approximating method for the higher powers.* Make several trials, until the number for the root, above and below the true root, is found. Involve the number below as the root to find its cube, &c., according to the power required. Call this root an *assumed root*—this power an *assumed power*.

Then say by the rule of three, as the sum of the *given*, and double the *assumed* cubes is to the sum of the *assumed*, and double the *given* cubes, so is the assumed root to the root required.

The fourth power being 67543, *to find the Root.*

17×17 three times gives 83521. This is above. 16×16 three times gives 65536. This is then the first below.

First Operation.

The assumed power, or the power of the assumed root 16, is 65536.

65536 assumed.		67543 given.	
2		2	
131072		135086	
67543 given.		65536	assumed power of root 16.
198615	:	200622	:: 16 : 16.1616

Second Operation.

67038.79		67543	
2		2	
134077.58		135086	
67543.		67038.79	assumed power of 16.16+
201620.50	:	202124.79	:: 16.16 : 16.2

Third Operation.

68874.75		67543	
2		2	
137749.5		135086	
67543.		68874	assumed power of 16.2
205292.5	:	203960.75	:: 16.2 : 16.09

As far as these operations are carried on, the nearer they approach the true root. But these alternate above and below the truth; and the medium is not to be considered as the truth. When the root is supposed to be nearly found, the proof is shewn by involution.

But all evolutions of the powers above the cube are long and tedious.

TRIGONOMETRY.

SEC. 31. ANGLES, or corners, are:

1. *Right angle*, square corner.
2. *Obtuse angle*, larger than a square corner.
3. *Acute angle*, smaller than a square corner.

SEC. 32. TRIANGLES, or three-sided figures:

1. *Right-angled triangle*, having one right angle. Its perpendicular line may be called the *vertical leg*—its bottom line may be called the *horizontal leg*.

2. *Obtuse-angled triangle*, having one obtuse angle.

3. *Acute-angled triangle*, having all the angles acute.

4. *Isosceles triangle*, having two of the sides of equal length; consequently, having two equal angles.

5. *Equilateral triangle*, having all the sides of equal length; consequently all the angles equal (just 60 degrees each.) This triangle necessarily includes the *isosceles* triangle.

6. *Scalene triangle*, having no two sides of equal length. The three first named kinds of triangle may be scalene.

SEC. 33. *Five miscellaneous items*, which should here be noticed by the student. They will be farther illustrated.

1. *A trapezoid* is any four-sided figure with only two parallel sides. Any figure bounded by right lines may be cut into trapezoids by latitude and departure lines [to be explained further on.] And the superficies of each trapezoid may be found by adding the parallel sides, multiplying their sum by their distance from each other, and halving the product.

2. *Equal triangles*. *Two lines are parallel*, when equi-distant from end to end; and all possible triangles, made between them on the same base, contain equal superficies, or areas. Any figure bounded by right lines may be reduced to a single triangle by the application of this principle. [To be explained further on.]

3. *Degrees of a circle*, are 360; each quarter or quadrant containing 90 degrees. The sine of an angle is a line let fall from one end of the arc of the angle, perpendicularly upon the opposite side. As the line *a c* is the sine of the angle *e*.

4. *Line of chords,* is a graduated line connecting the two ends of a graduated quadrant by a chord line. It is made by setting one foot of the dividers at *a*, and extending the other foot to each degree on the arc, and turning it down to the chord line. The chord line is used in plotting or projecting, when geometrical calculations are to be made.

5. *A triangle contains* 180 *degrees;* for a half circle contains 180 degrees, which is represented by the semi-circle *g h i*. By inspection, without taking the steps of a demonstration, the reader will perceive that the angles *a* and *a* are equal, also *c* and *c*, also *e* and *e*. Now as the angles *a c e* below the line *g i*, are measured by the semi-circle *g h i*, it follows that the angles *a c e* in the triangle above the line *g i*, may be measured by the same; consequently, contains 180 degrees. The same elucidation may be given of all forms of the triangle.

6. In every right-angled triangle, the long oblique side is called the *hypothenuse*. The other two sides are called the legs. And it may here be shewn the student, that the square of the two legs is equal to the square of the hypothenuse. This is a very important principle in practical trigonometry.

Sec. 34. *Geometrical trigonometry* gives all the sides and all the angles of a triangle, if two angles and one side, or two sides and one angle, are previously taken by the proper measures and observations.

From the following exemplification of this proposition, a reader of ordinary ingenuity, with no previous knowledge of trigonometry, may make all the applications which the following treatise requires. Draw the lines and angles given, then finish out the triangle in the only way in which it can be completed, without any random operation. The sides 70, 80, in the figure, and the angles at them are given. Draw the given line fifty feet, calling any division of a scale a foot. Strike the arc 80, *d*, with a radius of 60 degrees, taken with the dividers from the line of chords on Gunter's scale. Take 70 degrees, being the angle at 70, from the same line of chords, and set it off, on the arc, which will extend to *d*. Draw the line from 70 through *d* indefinitely. Then with the sweep of 60, again strike the arc 70, *f*, and set off 80 degrees upon it, being the angle at 80, which will extend to *e f*. Draw the line from 80 through *f* indefinitely, and it will cross the line which was drawn through 70, at *g*.

Where these lines intersect each other, is the true place for the other angle. Measure the two new sides by the same scale by which the given line was laid down, and you have all the sides. Add the two given angles together, which will make 150 degrees. Subtract this sum from 180, the degrees always contained in a whole triangle, and the remainder will be 30, the degrees of the new angle at *g*.

SEC. 35. *Proportions of sides and angles.* The angles of all triangles are proportioned to their opposite sides; therefore when two angles and a side, or two sides and one angle, are given, the other angles and sides may be found by the common rule of proportion, (rule of three.) But sines (as represented in number 3 of Sec. 33, *a c*) are used instead of degrees. These sines stand in tables, according to their actual lengths—calling the sine of 90, one.

SEC. 36. Trigonometry is a most sublime application of Mathematics. By it we learn the distances and movements of celestial bodies, which are millions of miles distant from us. And by it, also, we ascertain powers and movements, essential to our daily duties and comforts. Trigonometry, after all, owes its mighty powers to those laws, by which we ascertain two unknown sides of a three sided figure, by having but one side measured. The adept in mathematics knows, that the science requires that its applicants should be able to find one side from two, when the triangle has a square corner in it—or so that the sharpness and bluntness of a corner, made by the meeting of two sides, is necessary to most calculations. But these may be ranked with other principles and operations; the peculiar essentials being those just stated.

SEC. 37. A corner (always called an angle in treatises,) is best measured by a circle, struck around it with compasses, (usually called dividers,) with one foot standing exactly in the angle. The piece of the circle between the lines making the corner, is the measure of the angle. Every circle has always been divided into 360 degrees. Therefore, if one quarter of the circle, or 90 degrees of it, is included between the lines, the angle is a square corner, or 90 degrees—so of any other-sized angle. The circle is preferred as a measure; because a circular instrument for measuring has the most universal application. Students must learn the use of the compass, quadrant, sextant, &c., by actual shewing only.

Sec. 38. The two great principles applied to trigonometry are:

1. Application of the *square root* to the *theorem*, that in every triangle which has one square corner in it, (called a right-angled triangle,) the square of the slant-side (called hypothenuse) is equal to the square of both of the other sides—they being first squared separately and then added together.

2. Application of the *rule of three* to the theorem, that sides of triangles are proportioned to their opposite angles. That is, the greater the angle, the longer the opposite side, (meaning the side which does not touch the angle.) But this proportion is not direct, as 6 is to 18, so is 10 to 30, &c. But a table of sines, representing degrees, must be used instead of degrees. Students must be taught the use of the table of *natural sines*, by shewing only.

NATURAL SINES.

Sec. 39. It is not necessary to use tangents or secants in the application of trigonometry to the Mathematical Arts. Sines are sufficient in all cases. A table of natural sines is essential; but artificial sines and logarithms are not necessary. In some long series of calculations, particularly in working traverses of numerous courses, logarithms are a relief in the multiplications and divisions. Traverse tables, even then, are preferable.

The student should always be fully instructed in trigonometry by the use of natural sines *only*. Afterwards, he will learn the use and application of tangents, secants, and logarithms, in two or three days, if required.

Exhibition of Natural Sines. (*See figure.*)

Take the triangle *e s a*. The angle near *s* is 30 degrees. The sine of 30° is *a e*. The co-sine of 30° is *e x*—which is equal to *a s*, and *a s* is considered and calculated as the co-sine of 30°. Now call the radius one inch, and the sine *a e* will be half an inch. Look in the table of natural sines, and you will see radius (sine of 90°) 1.00000, and the sine of 30°, 0.50000, and the co-sine 0.86603. On measuring, all will be found thus to agree in measure with the table of sines, calling radius one. The same will hold true with every degree, minute, and second.

Directions for using the Abridged Table of Natural Sines.

Sec. 40. The sine of every degree and of every half degree is set down in the abridged table. In all full tables the sine of every minute of each degree ° ′ is included. For such tables the student is referred to the numerous books in use; but they are too long for this treatise. A common scale may contain this abridged table; and it will be found sufficiently accurate for all the common cases in practical Engineering, and in the Mathematical Arts in general. One minute of a degree will be the greatest error; and in all cases below 45 degrees it will never deviate a minute. Consequently, in all traverse cases it is sufficient, with the aid of the square root; and a useful substitute in all cases where full tables are not at hand.

Sec. 41. As only the sines of all degrees and of all half degrees are given, the minutes of any half degree must be found, as follows: Take the marginal figure as the augment (or increasing number) for each minute in the degrees following it. This added to the last preceding given sine, will give the sine of the degree and minute required. Thus: The sine of 14° 47′ is required. The sine of 14° 30′ is set down as .25038—the last sine preceding the sine required. The marginal figure for the augment of all minutes of all the degrees from 10 to 17 inclusive, is 28. As 17′ are to be added, 28 times 17 must be added to .25038 (the sine of 14° 30′.) 28× 17=476—this number 28 (the augment for each minute,) being added, makes .25038+476=.25514. The true sine of 14° 47′ is .25516—but this error cannot throw the sine into either the minute above or the minute below. For the sine of 14° 48′ would require 35, and the sine of 14° 46′ would be deficient by 26. If seconds of a minute are required, divide the augment by 60, and the quotient will be the addition to be made to the sine.

Sec. 42. When the middle term in the rule-of-three operation, is in sines, the answer will, of course, be in sines. To find the true minute to the sine thus obtained, proceed as follows: Subtract from the answer the sine of the degree, or half degree, next less than the answer. Divide the remainder, thus obtained, by the marginal figure (or augment) set against the degree. The quotient will be

the number of minutes to be added to the said given degree or half degree.

ABRIDGED TABLE OF NATURAL SINES.

Aug-ments.	De-grees.	S. C-S	C-S. S.	De-grees.	Aug-ments	Aug-ments.	De-grees.	S. C-S.	C-S. S.	De-grees.	Aug-ments.
29	0.00	0.00000	1.00000	90.00			23.00	.39072	.92050	67.00	11
	.30	.00873	.99996	.30			.30	.39875	.91706	.30	
	1.00	.01745	.99985	89.00	¼		24.00	.40674	.91355	66.00	
	.30	.02618	.99966	.30			.30	.41469	.90996	.30	
	2.00	.03490	.99939	88.00	¾	26	25.00	.42262	.90631	65.00	12
	.30	.04362	.99905	.30			.30	.43051	.90259	.30	
	3.00	.05234	.99863	87.00	1		26.00	.43837	.89879	64.00	
	.30	.06105	.99813	.30			.30	.44620	.89493	.30	
	4.00	.06976	.99756	86.00			27.00	.45399	.89101	63.00	13
	.30	.07846	.99692	.30			.30	.46175	.88701	.30	
	5.00	.08716	.99619	85.00	2		28.00	.46947	.88297	62.00	
	.30	.09585	.99540	.30			.30	.47716	.87882	.30	
	6.00	.10453	.99452	84.00			29.00	.48481	.87462	61.00	14
	.30	.11320	.99357	.30			.30	.49242	.87036	.30	
	7.00	.12187	.99255	83.00	3	25	30.00	.50000	.86603	60.00	
	.30	.13053	.99144	.30			.30	.50754	.86163	.30	
	8.00	.13917	.99027	82.00			31.00	.51504	.85717	59.00	14½
	.30	.14781	.98902	.30			.30	.52250	.85264	.30	
	9.00	.15643	.98769	81.00	4		32.00	.52992	.84895	58.00	
	.30	.16505	.98629	.30			.30	.53730	.84339	.30	
28	10.00	.17365	.98481	80.00			33.00	.54464	.83867	57.00	15½
	.30	.18224	.98325	.30			.30	.55194	.83389	.30	
	11.00	.19081	.98163	79.00	5	24	34.00	.55919	.82904	56.00	
	.30	.19937	.97992	.30			.30	.56641	.82413	.30	
	12.00	.20791	.97815	78.00			35.00	.57358	.81915	55.00	16¼
	.30	.21644	.97630	.30			.30	.58070	.81412	.30	
	13.00	.22495	.97437	77.00	6		36.00	.58779	.80902	54.00	
	.30	.23345	.97237	.30			.30	.59482	.80386	.30	
	14.00	.24192	.97030	76.00		23	37.00	.60182	.79864	53.00	17
	.30	.25038	.96815	.30			.30	.60876	.79335	.30	
	15.00	.25882	.96593	75.00	7		38.00	.61566	.78801	52.00	
	.30	.26724	.96363	.30			.30	.62251	.78261	.30	
	16.00	.27564	.96126	74.00			39.00	.62932	.77715	51.00	18
	.30	.28402	.95882	.30			.30	.63608	.77162	.30	
	17.00	.29237	.95630	73.00	8	22	40.00	.64279	.76604	50.00	
	.30	.30071	.95372	.30			.30	.64945	.76041	.30	
27	18.00	.30902	.95106	72.00			41.00	.65606	.75471	49.00	19
	.30	.31730	.94832	.30			.30	.66262	.74896	.30	
	19.00	.32557	.94552	71.00	9	21	42.00	.66913	.74314	48.00	
	.30	.33381	.94264	.30			.30	.67559	.73728	.30	
	20.00	.34202	.93969	70.00			43.00	.68200	.73135	47.00	20
	.30	.35021	.93667	.30			.30	.68835	.72537	.30	
	21.00	.35837	.93358	69.00	10	20½	44.00	.69466	.71934	46.00	
	.30	.36650	.93042	.30			.30	.70091	.71325	.30	
	22.00	.37461	.92718	68.00			45.00	.70711	.70711	45.00	
	.30	.38268	.92388	.30			.30	.71325	.70091	.30	

Illustrations of Trigonometry.

The different modes of applying the Square Root and the Rule of Three, to cases of Trigonometry, are *five.**

Sec. 43. *Any two sides of a right angled triangle being given, the other side may be found by the square root, without the aid of angles.*

1. *Illustration.* If the height of a wall is known to be 20 feet, and a ditch at the bottom of the wall is 12 feet wide; the length of a ladder for scaling the wall may be found, by squaring the height of the wall and breadth of the ditch separately, adding the two products and extracting the root of the sum:

20	12	400	544.0000(23.32 Answer.
20	12	144	4
400	144	544	43)144
			129
			463)1500
			1389
			4662)11100
			9324

2. *Illustration.* If the width of the ditch and length of the ladder are known, the height of the wall may be found by squaring the length of the ladder and the breadth of the ditch separately; then subtracting the square of the breadth of the ditch from the

* In all cases, a student ought to plot a triangle, before commencing the calculation. An old experienced engineer can obviate numerous perplexities, by making hasty random sketches of whatever figures he may attempt to calculate.

square of the length of the ladder, and extracting the root of the remainder.

23.32	12	543.8224 sq. ladder.
23.32	12	144 sq. ditch.
4664	144	399.8224(19.99 Answer.
6996		1
6996		
4664		29)299
543.8224		261
		389)3882
		3501
		3989)38124
		35901
		2223

Loss by omitted fractions reduced 20 feet, one hundredth of a foot.

SEC. 44. *One side of any triangle, and two of the angles, being given; the other angle may be found by subtraction, and the other two sides by the rule of three with sines of the angles.*

1. *Illustration.* The distance between a corner of the school-house and the steeple of the church is required. An imaginary triangle is so made, that the steeple is in one angle, the corner of the school-house in another, and a stake is set up in the yard, 80 feet from the corner of the school-house, for the other. The compass, or quadrant, is set at the corner of the school-house; by which it is found, that the imaginary lines to the church and stake form an angle of 80 degrees at this corner. The compass is then set at the stake; by which it is found, that the imaginary lines to the church and school-house, form an angle of 70 degrees at the stake. As every triangle contains 180 degrees, on subtracting the 80 and the 70 degrees (that is, 150) from 180, it will leave the angle at the church 30 degrees. We shall thus have obtained one side of a triangle 80 feet, and all the angles, 80, 70, and 30. We

then state—if 30 degrees (at the church) give 80 feet, (from the school-house to the stake) what will 70 degrees (at the stake) give? But, as sines of degrees must be used instead of degrees, the operation is as follows:

As the sine of 30° (at the church) is to 80 feet, so is the sine of 70° (at the stake) to the distance from the school-house to the church.

	30°	:	80	:: 70°
Sine	.50000			Sine .93969
				80
[brought up.]			417520	)84.17520(168.3 Answer.
			400000	50000
			175200	341752
			150000	300000
	Remainder		25200	41752

We find the distance between the school-house and church to be 168 feet and three-tenths of a foot, with an immaterial remainder.

The distance between the stake and the church may be found by the same process, taking the sine of the angle at the school-house for the first number.

NOTE. Future examples and descriptions will be less minute on the application of the *rule-of-three* and *sines* to Trigonometry; therefore the student must work out numerous examples (furnished by the teachers, or from other books,) under this mode of application.

2. *Illustration.* The distance from the centre of the earth to the moon is required. Imagine a triangle, of which the moon is at one angle, the centre of the earth at another, and a place where you stand is at another. The angle where you stand is 90 degrees; because the sensible horizon is one side of the triangle, and a perpendicular line to the centre of the earth is another. The angle at the moon is $57\frac{1}{6}$ minutes of a degree (as found by the parallax, hereafter to be demonstrated.) We then state: if $57\frac{1}{6}$ minutes of a degree give 4000 miles (the distance, near enough, to the centre of the earth,) what will 90 degrees give? Thus:

	$57\frac{1}{6}$	:	4000 m.	::	90°
Sine	.01663		1.00000		1.00000 Sine.

.01663)4000.00000(240,529 Answer.
3326

6740
6652

8800
8315

4850
3326

15240
14969

The distance of the moon from the centre of the earth, is generally set down at 240.000 m. I have always made it a little more.

Sec. 45. *If two sides of any triangle are given, and one of the angles which is opposite to one of the given sides ; the other two angles and the other side, may be found by the rule of three and sines.*

Illustration. We are on the east hill, and wish to ascertain the length of a level air-line to the west hill, at an equal height. The two adjoining sides are straight, smooth, and convenient for chaining; and meet at a well defined angle at their bases. The east side hill has a slope of 196 feet—the west side hill has a slope of 360 feet—the angle made on the east hill by its slope and the horizontal level is 72°. We state as follows :

As 360 feet : to 72° : : 196 feet to the angle on the west hill.

360 f. : 72° : : 196 f. : .51777=31° 11′

```
          Sine .95106
                  196       )186.4076(.51777=31° 11′
               ------        1800
               570636        ----
              855954          640
              95106           360
              --------        ----
[carried up.] 186.40776        2800
                               2520
                               ----
                                2807
                                2520
                                ----
                                 2876
                                 2520
```

The angle on the east hill, 72°
The angle on the west hill, 31° 11′

The two angles equal 103° 11′
Subtract from 180°
The other two angles 103° 11′

Angle at the base, 76° 49′

Find the air-line thus:

```
72°   :   360  : :  76° 49′      350.51400(379 f. Answer.
.95106              .97365       275318
                       360       ------
                   -------        751960
                   5841900        6658.42
                  292095          -------
                  --------         862180
    [carried up.] 350.51400        855954
                                   ------
                                     6226  Rem.
```

SEC. 46. *If two sides of any triangle are given, and an angle formed at their meeting; the other two angles and the other side may be found by the rule of three, sines, and the square root.*

Illustration. A road was traversed with chain and compass, through an uneven parish. At one place the turn was so short, that it was necessary to strike a circle to cut through three of the corners (being at the meeting, and ends, of two of the surveyed lines.) They run thus: north 44 degrees east 8 chains 50 links; and north 60 degrees east 12 chains 40 links. By drawing a north-and-south line through the place of their meeting on the plot, and reversing the first line (calling it south 44 west) and subtracting the 60 degrees of the second line, from 180, the meeting angle will be found to be 164°. Imagine the first line extended beyond the angle so far, that a perpendicular, let fall from the end of the second line, will strike its termination. Then the first line, ideally extended, and the imaginary perpendicular, will form the two legs of a right-angled triangle. The side sought for, connecting the extreme ends of the two surveyed lines (which is to be the chord line of the proposed arc,) will be the hypothenuse; and is found by the square root, in the usual way. As these two ideal lines and the second known side, form a right-angled triangle, they are found in the usual way, by the rule of three and sines—the 164° being subtracted from 180°; leaving the adjoining angle (within the new triangle) 16°. Thus:

$$\begin{array}{l}180° \\ 164° \\ \hline 16°\end{array} \text{ one acute angle.} \qquad \begin{array}{l}90° \\ 16° \\ \hline 74°\end{array} \text{ the other acute angle.}$$

$$\begin{array}{lcccr}
90° & : & 12.40 & :: & 16° \\
1.00000 & & & & .27564 \\
 & & & & 12.40 \\
\hline
 & & & & 1102560 \\
 & & & & 55028 \\
 & & & & 27564 \\
\hline
 & & & & 3.3179360
\end{array}$$

the perpendicular in chains, links, and decimals.

90°	:	12.40	:: 74°
1.00000			.96126
			12.40
			3845040
			192252
			96126
			11.9196240 the hori-

zontal leg in chains, links, and decimals.

Add the horizontal leg obtained, to the first surveyed side, as 8.50 +11.92=20.42 for the whole horizontal leg. Then find the chord line of the proposed arc thus:

20.42	3.32
20.42	3.32
4084	664
8168	996
40840	996
416.9764	11.0224
11.0224	

427.99 88(20.64+Chord line of the proposed arc; which arc is to strike the meeting angle and the extreme ends of the two surveyed lines.

```
    427.99 88(20.64+
    4
    -
406)2799
    2436
    ----
8124)36388
     32496
```

Note. This is a very important application of trigonometry in laying out rail-roads, McAdam roads, canals, &c., as will be shown farther on. The operation is often performed with a table of tangents; but I can perceive no advantage in driving the practical mathematician from the table of natural sines.

Sec. 47. *If one leg of a right angled triangle is given, and the sum of the hypothenuse and the other leg, the whole triangle may be completed by the square root, the rule of three, and sines.*

Illustration. A green-house (for the defence of plants which cannot brave our winters, but do not need the hot-house) was threatened by the fall of a decaying poplar, 22 feet south of its sash-glass roof, which extended to the ground. The top of the tree had been previously cropt, at the height of 64 feet. It appeared to be necessary, that the trunk should be cut, so as to fall at the exact south margin of the green-house. The proprietor of the green-house called on a neighboring practical mathematician, to ascertain how high from the ground this tree must be cut, so that (hanging by the bark and sap-wood as by a hinge) it should fall with its top just at the outer edge of the green-house sashes. His inquiries were answered as follows: Square the given base (22 feet,) and square the height of the tree (64 feet;) add the two squares, and extract the root, which will give the hypothenuse from the top of the tree to the margin of the sashes.

Consider this hypothenuse as one side of a right angled triangle; and find the angle at its top as in other cases of right angled triangles, where all the sides are given—it will be 18° 58′. Imagine the tree to be cut as intended, and the top fallen. As the supposed fallen part and the same part erect, are equal, the angles at the top and bottom are equal—that is, each is 18° 58′. Add these two angles together and subtract their sum from 180, which will give the middle angle of 142° 4′. Subtract the middle angle from 180, which will give the top angle of the triangle sought, 37° 56′. Then proceed as in all cases of a right angled triangle, with one side and one angle given. It should be remarked, that the top angle of the created or borrowed triangle, is not that of the sought triangle; consequently, that double the top angle produces the top angle of the sought triangle.

Other methods are given in books; but this is the most simple. This case is very important in conic sections; particularly the ellipse.

Sec. 48. *The three sides of any triangle being given, it may be completed, also its area found, by the square root, rule of three, and sines.*

Illustration. One rafter of an unequal roof is 40 feet, the other 20—the cross-beam is 50 feet. This oblique-angled triangle must be divided into two right angled triangles. This will require that a perpendicular be let fall from the meeting of the rafters, to the beam, so as to divide it into two parts, each according to the length of the rafter stretched over it, thus :

As the length of the beam 50 feet, is to the sum of the lengths of the two rafters (40+20) 60 feet, so is the difference of the two rafters (40—20) 20 feet, to the difference between the parts of the beam into which the perpendicular divides it.

50 :	60 ::	20	2)24(12 the half difference.
	20		
2)50	1200(24		2)50(25 half the beam.
—	100		12
25 h. b.	——		—
12	200		13 shortest half of the beam.
—	200		
37 longest half of the beam.			

Having obtained the base leg of each of the two right angled triangles; and the hypothenuse (rafters) of each being given, the perpendicular leg to both, and the acute angles of both, may be found as in all cases where the hypothenuse and one leg are given.

This rule may be applied in finding angles in fields, which had been measured without a compass or other instrument for taking degrees.

Sec. 49. Of the three angles and three sides of every triangle, *three* of these six constituents must be known, and one of the known constituents must be a side, excepting the corner where the square root applies. But in a right-angled triangle and in an isosceles triangle, one side and one angle only are to be taken in the field. Because every right-angled triangle contains one right angle, and every isosceles has two equal-angles. The isosceles triangle is of great use in rail-road curves and other curvilinear calculations.

Sec. 50. In figures of many sides, it is often convenient to know at sight, the sum of the degrees contained in all the angles, outer and inner. Count the angles—deduct two from the whole—multiply the remainder by 180. This is too evident from inspection to require illustration. Take a figure of 7 sides: $7-2=5\times180=900$ degrees.

Sec. 51. Examples *in all the cases; but they are not set down in systematic order.*

1. The height of a tree is required. The distance to the tree is 30 feet, the angle made by a line to the top of the tree, with a line to the bottom, is 30° 15′—the ground to the tree is level, and the tree is perpendicular.

2. A rope to the top of a tree is 62 feet long; the angle made by the rope and a line to the bottom of the tree, is 42°—the ground is level, and the tree perpendicular. How high is the tree?

3. Two stakes on the east side of the river, are 2 chains 64 links apart. The angle at the north stake, formed by the measured line between the stakes, and an imaginary line drawn to a cedar tree on the west side of the river, is 79°—the angle at the south stake, formed by said measured line, and an imaginary line drawn to said cedar tree, is 83°. How far is it across the river, from the south stake to said cedar?

4. A dam is 900 feet long. An imaginary line, drawn from a pine tree on the west shore of the river, to the east end of the dam, forms an angle of 68° with the line of the dam. How far is the pine below the west end of the dam, the shore being perpendicular to it?

5. Determine whether the outer base-sills of a house form perfectly square corners. Sills of the sides, 48 feet—of the ends, 37 feet. Find the diagonal by the square root. Then measure with a tape, and see whether the measured diagonal agrees with the calculation. If not, rack the whole base with a lever until it will agree.

6. The upper ceiling of a room is 11 feet 6 inches high, the floor is 32 feet 9 inches long. Find the length of a tape required, to reach from the upper ceiling at one end of the room, to the floor at the other end, by the square root.

7. The length of a brace is required from shoulder to shoulder inside, and from shoulder to shoulder outside—the centre of the mor-

tice, on the beam, is 7 feet from inner meeting of the beam with the post; and, on the post, 9 feet. Determine the length by the square root.

NOTE. The angle of the mitred shoulders of the braces are determined by a reference to the homologous sides between the shoulders and the right angled triangle, formed by the brace, post, and beam.

8. You take a station where you can see a flag on each side of the base of a mountain; you wish to ascertain the distance through the mountain from flag to flag. Find the distance from your station to both flags, by trigonometry or measure—also take the angle at the station. Then find the distance through the mountain, as in all other cases, where two sides and a contained angle are given, for finding the other side.

9. A straight line to be drawn obliquely through a block of houses is required. You can contrive to compass the block by a base line 140 feet long; and two other lines meeting at the point where the measure is to fall perpendicularly on the base, one 92 feet and the other 67 feet; but could take no courses, nor angles.

10. You purchased a piece of ground, which is to be in the form of a right angled triangle, with the base leg 20 chains. The sum of the perpendicular leg and the hypothenuse to be 60 chains. What will be the length of the perpendicular leg, what will be the length of the hypothenuse, and how much land will be included in the triangle?

MENSURATION*

IS DIVIDED INTO SUPERFICIES AND SOLIDS.

SEC. 52. *Superficial Mensuration.*

1. PARALLELOGRAM. A garden is 50 feet wide and 200 feet long. All the sides meet at right angles. It is manifest a strip from one end, that is one foot wide, contains 50 feet, another foot takes in 50 feet more, and so on. Therefore the whole 200 feet

* As Mensuration, when practically applied, requires more or less extended illustration, according to the peculiar circumstances of cases, I shall merely give the most common rules in use in few words—reserving for special applications such further demonstrations as may appear necessary.

contains 200 times the first 50 feet strip. Hence the rule: *multiply the length and breadth together and the product will be the square feet contained in a parallelogram or square.*

2. Triangle. If a diagonal line is drawn through the aforesaid garden, it will be divided into two equal parts. Consequently each part will be a triangle containing half as much as the whole garden. As the long side of each triangle, called the base, is 200 feet, and the short side, called the perpendicular, is 50 feet, it follows that if the base is multiplied by the perpendicular, the product will be double the true contents. Hence the rule: *multiply the base of a triangle by its perpendicular, and halve the product; which will give the square feet contained in it.*

3. Regular polygon. Take for example an octagon. It may be considered as consisting of eight isosceles triangles, whose apexes meet at the centre and whose bases constitute the eight sides. Now if all these eight bases are added together and that sum multiplied by one perpendicular, it is manifest that the product will be double the contents of all the eight triangles. Hence the rule: *multiply the sum of all the sides of a regular polygon by their distance from the centre, and half the product will be the area.*

4. Circle. Suppose the sides of the aforesaid polygon to be indefinitely short, so as to form a polygon of many million sides; it is manifest that the arc of a polygon, called a circle, may be found as of polygons with longer sides. Hence the rule: *multiply the periphery of a circle by half its diameter, and half the product will be the area.*

5. Periphery of a circle is produced by multiplying the diameter by 3.1416—more accurately, 3.14159.

6. Diameter of a circle is produced by dividing the periphery by 3.1416.

7. Area of a circle may be produced by squaring its diameter; and then displacing the corners by multiplying the product by .7854. For example, suppose the diameter of a circular garden bed 10 feet. Multiplied by itself the product is 100. Multiply the 100 feet by .7854, and the true product will be 78.5400 or—78 feet 54 hundredths.

8. Length of any arc of a circle may be found by multiplying the degrees of the arc, the radius, and the formula .01745, into each

other. Thus 40 degrees of an arc with a radius of ten inches will stand thus: $40 \times 10 \times .01745 = 6.98$ inches.

9. Sector of a circle may be found by multiplying the length of the arc (as found above) by the length of the radius, and halving the product. This is an application of the same principle which is applied to the regular polygon and circle.

10. Segment of a circle may be found by first finding the area of the sector and then subtracting the area of the triangle made with the chord line and two radii. But this triangle must be added, if the segment is greater than half the circle.

11. Area of an oval may be found as the area of a circle. That is, by bringing the oval to a circle by multiplying the longest diameter by the shortest, and that product by .7854.

12. The superficies of a prism or cylinder may be found by multiplying the pyrimeter of either end by the length, and adding the area of both ends.

13. The superficies of a pyramid or cone may be found by multiplying the pyrimeter of the base by the slanting side, and halving the product—to this add the area of the base.

14. The area of a parabola may be found by multiplying the base by the perpendicular and deducting one third of the product.

15. The superficies of a globe may be found by multiplying its circumference by its diameter. If the superficies of a segment is required, multiply the whole circumference by the height of the segment.

Sec. 53. *Solid Mensuration.*

1. The solid contents of a cube, parallelopiped, prism, or cylinder, may be found by multiplying the area of one end (or side) by the length.

N. B. A wedge may be considered as a triangular prism; the edge forming one angle of the prism, and the corners of the head of the wedge, as the other two angles of the prism.

2. The solid contents of a globe may be found by multiplying the surface by the diameter, and taking one sixth of the product for the solid contents.

3. The solid contents of a pyramid or cone may be found by multiplying the area of the base by the perpendicular, and taking one third of the product for the solid contents.

4. The height of a pyramid or cone may be found by the rule of proportion, if the height and upper and lower diameters of any frustum of it is given, thus: as the difference between the diameters is to the height of the frustum, so is the upper diameter to the height of the part above the frustum. If this be added to the height of the frustum, the sum will be the height of the whole pyramid.

5. The solid contents of the frustum of a pyramid may be found by finding the height of the part above the frustum, as directed in the last rule; then finding the solid contents of the whole pyramid, and of the part above the frustum, as before directed, and subtracting the latter from the former—the remainder will be the solid contents of the frustum. The same rule applies to the *cone;* but after the above process is finished, the said remainder must be multiplied by the formula, .7854. Example: Lower diameter of the given frustum is 10 inches, upper 6, height 8. As dif. 4 : 8 :: 6 : 12. Height 8 and 12=20. 10×10=100×20=2000. 6×6=36 ×12=432. 2000—432=1568 divided by 3=522.66 Answer. Contents of pyramid multiplied by .7854=410.49 Answer.

6. The solid contents of the frustum of a pyramid* may be found by multiplying the breadth of the top by the breadth of the bottom, and multiplying that product by the height. To the last product add a sum, produced by squaring the difference between the breadth of the top and bottom, and multiplying that square by one third of the height.

If it is the frustum of a cone, the last product alone must be multiplied by the formula, .7854.

7. Guaging. The parts of a cask on each side of the bung are frustums of cones, as A B, A B, (see figure) are frustums of A C, A C, considering the staves as straight from the bung to the heads; therefore their contents are found in the same way, separately. By doubling them, the contents of both frustums, or the whole cask, is found.

* I have never seen this method published. As far as I know, it was first used at the Rensselaer Institute. It may be demonstrated by a model.

But the staves generally curve more or less; for which an allowance must be made, unless the convexity of the inner surface of the heads occupies a space equal to what is added by such curviture. Common casks require that about a tenth part of the difference between the head diameter B and the bung diameter A, be added to the bung diameter A. This will increase the contents so as to equalize the contents of the curvitures *e e e e*.

For expeditious practice in guaging, graduated *guaging rods* are used. Or the following rule and formulas may be adopted.

8. Add the head and bung diameters A and B (taken in inches) and take half that sum for the average diameter. To this average diameter add a sum equal to one eighth of the difference between the two diameters, for the curviture of the staves *e e e e* in common casks. Square the last sum and multiply it by the length of the cask B B. As this gives too many square inches, divide the product as follows: if for wine of 231 inches to the gallon, divide by 294.12—if for beer, cider, ale, &c., of 282 inches to the gallon, divide by 359.05. The quotient in both cases will be in gallons. If for bushels of 2150.4 inches to the bushel, divide by 2738.

9. The tonnage of a ship, according to a statute of the United States, must be found as follows: Take the length of the vessel from the fore part of the main stem to the after part of the stern post above the deck or decks; and the breadth at the broadest part above the main wales, and deduct from the length three fifths of the breadth.

For a double-decked vessel, half of the breadth shall be accounted the depth; then the length, breadth and depth must be multiplied together, and the product divided by 95—the quotient will be the tonnage.

For a single decked vessel, the depth must be taken from the under side of the deck plank to the ceiling in the hold. In all other respects proceed as with double deckers.

Ship carpenters measure is made by proceeding as above in all respects, excepting that they take the length of the keel, the breadth of the main beam, and the depth of the hold; though in double-deckers they take half of the breadth for the depth as before stated, under *double-decked* vessels.

LAND SURVEYING, (*Geodesia,*)

IS DIVIDED INTO FOUR KINDS.

Sec. 54. (*Pediometry.*) Field surveying. This is applied by farmers, when the contents of a *field* are required for the purpose of ascertaining its productiveness by the acre, for determining the quantity of seed to be sown, or to estimate the value of labor in ploughing, mowing, harvesting, &c.

Sec. 55. (*Agrometry.*) Farm surveying. This is applied when the out-bounds, courses, distances, and contents, of a *farm* or *lot* are required, for the purpose of description in a deed, for the establishment of boundaries, for ascertaining the contents, for making a map, &c. For this kind of survey the courses and distances are not given.

Sec. 56. (*Orometry.*) Line surveying. This is applied when the obscure or lost lines of a previous survey are to be revived and marked; or where lines are given, by a mere plot upon paper, to be traced and marked. For this kind of survey, the courses and distances are given, which are taken from the previous survey, or from a measurement made with instruments on the paper plot.

Sec. 57. (*Udrometry.*) Aquatic surveying. This is applied to harbors, rivers, lakes, ponds, marshes, &c., where the measure of surfaces, the location of shoals, depths of water, quality of the underlaying earth, &c., are required, in places which cannot be approached on dry land in the usual way.

Sec. 58. These four names are compounds of the Greek *metreo* (to measure) with the following Greek words: 1. *Pedion* (neuter) a plain, open, level field. 2. *Agros* (masculine) ground, land, a farm. 3. *Oros* (masculine) a boundary, a land-mark. 4. *Udor* (neuter) water. These derivations are given to aid the memory of the student; while the names will assist him in systematizing his views of practice.

1. PEDIOMETRY. (*Field surveying.*)

Sec. 59. As any figure bounded by straight lines, can be cut into triangles, and as the contents of any triangle may be found by multiplying its base by a perpendicular line, drawn in the nearest direc-

tion from it to the opposite angle; it follows, that the contents of any open field may be found by cutting it into triangles.

SEC. 60. *Use of the cross.* In the figure, suppose the dotted lines to represent imaginary lines in the field, actually measured thus. Measure with a rope or chain from corner to corner, as from B to H—from H to G—and from D to F. After measuring these lines, take the perpendicular, with a cross. The cross may be made by laying two laths or strips of board across each other, and nailing their centre to the top of a staff. With the cross move back and forward, until the point is found, where one slip of the cross points to the angle, as at A, while the other lines point at the two ends of the base line, as at B and H, then measure the perpendicular. In this way take the base and perpendicular of every triangle, and multiply each base by its perpendicular. Add all the products, and halve that sum; which will give the contents of the field in the squares of whatever measure was used—if the rope was in feet, the answer will be in square feet; if in rods, in square rods; if in chains, in square chains.

SEC. 61. *Field calculation.* If the number of acres are required, and the measure was taken in chains and links, the calculation is very simple. Example. Base B H 12.30, perpendicular to A 7.21, and perpendicular to C 8.42. These two perpendiculars may be added together before multiplying; because they have the same base. Then $7.21+8.42=15.63\times12.30=192.24$. Base H G 11.05 per. to C 4.00 multiplied together $=44.20$. Base D F 11.96 per. to C 10.21 and per. to E 7.15; which perpendiculars added make 17.36 multiplied by $11.96=207.62$. Add the products of all the triangles $192.24+44.20+207.62=444.06$. Half of that sum, 222.03 is the contents of the field in square chains and links. As ten square chains are an acre, divide the chains by 10, which gives 22 acres. Multiply 2.03 by 4 and divide by 10, which gives 32 rods. The answer is 22 A. 0 Q. 32 R. omitting all fractions below links. But if there are numerous triangles and accuracy is required, the fractions must be retained until the operation is finished, and merely rejected from the final answer.

SEC. 62. *Compass necessary.* The student will perceive, that maps cannot be so drawn from such surveys as to give the points of compass; neither can deeds be drawn by them. The next

method must be resorted to in all such cases; also in fields where hills, woods, &c., obstruct the sight.

II. AGROMETRY. (*Farm surveying.*)

Sec. 63. *Remark.* This being the most important kind of surveying, and that to which all the other kinds will be referred for their chief explanations; I shall give directions in a most familiar manner. I can devise no plan more eligible, than that of arranging the directions under distinct heads; so that the whole shall form the history of a survey.

History of an Agrometric Survey.

Sec. 64. Having received an application to survey a farm owned by two brothers; and to divide the same, unequally, as to quantity, between them, I proceeded as hereafter related. This survey is a real case in my practice; excepting that I took in parts of three other real surveys, for the sake of making it more diversified.

Sec. 65. *Magnetizing the needle.* With a view to execute the job with accuracy, with a strong magnet I retouched the needle of my compass. As magnets communicate contrary poles and attract contraries, the only safe method of touching the needle is, to bring it near the magnet, suspended on a pin's point as a pivot, and let its poles choose for themselves. Then rub the pole of the needle on the end of the magnet it has chosen; and the other pole on the other end.

Sec. 66. *Correcting chain.* I remeasured my chain, and added a few wire rings to bring it to the precise measure of 66 feet; and also to equalize all the links—carefully counting the hundred links, and equalizing the quarters (or twenty-fives) into rods, and seeing that the whole chain was accurately tallied off in tens of links.

Sec. 67. *Stakes and tallies.* I counted over my nine wire stakes, and supplied the deficiencies; having each 12 inches long, with a well-turned eye at the top. In each eye I tied strips of red, white, and black rags; that they might be seen readily among dead leaves, evergreens, &c. I made a new set of 7 leather tallies; which consisted of inch-square pieces of sole-leather, strung upon a

cord passed through their centres. The cord was of a sufficient length to pass around the waist of the hind chain-bearer.

Sec. 68. *Field book, pens and ink.* I prepared a blank book, four inches square, covered with thin leather; fitted to carry in the left breast-pocket. And I sewed a slender vial in the fore edge of the same pocket, containing good ink, absorbed by cotton wadding. One short pen was set into the vial; and several other short quills were fastened in the bottom of the same pocket.

Sec. 69. *Scale, dividers, protractor and ruler.* Next I screwed up the joint of my best dividers, and made the points sharp and smooth as the points of sewing needles. Selected my best ivory scale, and most accurate semi-circular protractor—also, a wooden ruler, one inch wide, with its opposite edges precisely parallel, straight, and thin; being supported in a perfectly straight form by a high ridge along the centre of the upper side. This ruler is for drawing random parallel meridians, by its opposite edges.

Sec. 70. *A protractor for laying parallels.* My protractor being of the common form, I ground down the upper side of the straight part to a sloping bevel, and engraved a line along the face of the bevel, near its edge, and exactly parallel to it. This line is for setting in a foot of the dividers, when laying the straight side of the protractor parallel to a meridian line.

Sec. 71. *Taking elevations and depressions, and moderate heights.* My employers did not belong to that penurious class, who prefer a hurried, half taken survey, to expending another dollar to gain fifty in utility. Therefore, I prepared for taking all the inequalities of surface along the line, worthy of notice; also the heights and distances of important buildings, &c. A very simple and cheap instrument is Kendall's tangent-scale. This is prepared by striking a quadrant with a radius equal in length to the distance between the outsides of the sights. Make a tangent to this quadrant, and set off the degrees and half-degrees upon a scale; rather upon the edges of the sights, to be continued on a scale when required. By ranging upwards from the bottom of the slit in the opposite sight, or ranging

downwards from the tangent scale, through the same, ascents and descents may be taken with sufficient accuracy.*

I put up my brass slip for reducing the farm to a single triangle, which will be described in its proper place, with its application.

Sec. 72. *Assistants.* On the morning appointed, I repaired to the farm to be surveyed. I found all the assistants in readiness as I had directed. One was selected for *hind chain-bearer;* because he had more talents and learning than the rest. On the accuracy of the hind bearer depends the correctness of the chaining. He directs the fore bearer so as to keep him in range with the flag, he receives all the nine stakes, wears the string of leather tallies, and slides one for every ten chains, and renders the account of chains and links when called on by the surveyor. His pay was one dollar and twenty-five cents per day. The *fore chain-bearer*—pay fifty cents per day. *Ax-man*—pay fifty cents per day. *Baggage-man*—pay fifty cents per day. *Flag-man*—pay seventy-five cents per day; for his duty, though very light, requires considerable judgment and great care.

Sec. 73. The first step taken at the farm was to go around it, and put up a stake at every place where the line turned sufficiently to change the course. I had advised my employers to have their interested neighbors present, to see the boundaries fixed, to avoid future controversy. They all attended us and all the men employed, except the baggage-man. Him I directed to cut a perfectly straight flag-staff, 10 feet long, and mark it off into feet, and wind spirally a red, white, and black cloth, around three feet of its upper end. And advised my employers to have put up in a pack or basket,† such articles of refreshment as we might need, in order to save the time, which would be consumed while returning to take formal meals.

* This simple operation was suggested to me in the year 1831, by Mr. Thomas Kendall of New Lebanon. Its utility has been tested sufficiently for five years at Rensselear Institute. Hanks' compasses are furnished with an appendage, well adapted to this, and other useful purposes. Meneely has revived the method which was approved and in common use about the middle of last century. But it is the cheapness and universal application, that gives Kendall's method its value.

† In surveys of new lands, where the party sleep in woods, the baggage-man must carry an oil-cloth to spread under, and a tent to stretch over, the whole company. A poney, with a leather cover sewed to the fore end of the saddle and spreading back over all the baggage, to defend it from storms and from being scratched and torn, is useful.

All the other men went around with us to cut and set up stakes, to throw stones about the principal corners ; the flag-man was directed to take particular notice of the corners, so that he might readily find them.

Sec. 74. *Fixing first corner.* Having set up stakes at all the corners, we entered upon the survey. As the employers were to exchange deeds of release, which might lay the foundation for future subdivisions among heirs and purchasers, as well as between the present owners, I was careful to fix the boundary for the place of beginning by reference to objects which might be found in future, as follows :

Beginning at a stake and stones, standing on the east bank of Stoney Brook, at a place thirteen chains from the junction of said brook with Meadow Brook on a course N. 10 W. and eleven chains below Griswold's mill, on a course, along said brook, S. 23 W.

Sec. 75. *Taking the course.* I set my compass-staff into the ground at the said stake, marked 1 in the map. Having sent the flag-man to the corner marked 2, I directed the sight of the compass to the flag. Observing that the forward sight was nearer the north end of the needle than the south, I set N. in my field book ; and as the same sight was on the east side of the needle, I set E. in my field book ; and as its distance east was $43\frac{3}{4}$ degrees, I set this number between the said letters—thus, N. $43\frac{3}{4}$ E.*

Sec. 76. *Compass staff.* As the wind blew with considerable violence, I found my compass staff was too slender to keep the compass steady. I directed the axe-man to fit a stiffer one into the iron socket at the bottom, and the brass one at the top ; which he did with an interruption of but thirteen minutes of time. My staff was now just such as I always preferred. It consisted of a straight hickory sapling, two inches in diameter in the middle, with the bark on it, and four feet long between the sockets. From the lower end of the upper socket to the bottom of the sights, was six inches ; and the iron socket at the bottom, including its strong steel beak, was twelve inches. This is a suitable length for a surveyor six feet in height. And its great weight enables him to strike it firmly into

* A correct surveyor always makes N. or S. the leading course, unless the course is due east or west; in which case the word is spelled out. He always says *north* or *south*, so many degrees east or west; and plots by north and south, or meridian lines.

the ground. A tripod will never be used in the field by an experienced surveyor.

Sec. 77. *Directions to chain-bearers.* I then gave my directions to the chain-bearers, as follows:

1. That neither of them should ever pass the compass, when set for taking the course.

2. That the fore-bearer should carry all the stakes in his left hand but one, and that in his right hand, clenched around the stake with the chain-ring, and its point in the direction of the thumb.

3. That he should never look back, but walk on with his eye upon the flag, until the hind-bearer cries *down;* then he should set the stake firmly, and cry *down*, without looking back, and lead the chain about three inches to the left of the stake, to avoid dragging it from its place.

4. That he should continue in the same manner, unless he is stopped by the flag-man, until his nine stakes are out, and until he has stretched his chain and finds he has no stake to set—then he cries *tally*.

5. The hind-bearer, having carefully arranged the fore-bearer by the flag, and having received all the stakes, at each of which he cried *down;* now should drop his chain-ring, and walk to the fore-bearer. But he must never in any case carry forward the hind ring or in any way double the chain.

6. On delivering the stakes to the fore-bearer, both must count them.

7. The hind-bearer then slides one of the leather tallies from the left side of the knot tied in the cord behind his back, around to the right side of the knot, and set the fore-bearer on his way again, holding the toe of his shoe precisely on the mark where the exchange of stakes was made, until the hind ring comes up to it, when he cries *down*. Then all must go on as before.

8. That if the fore-bearer comes to the flag between tallies (which will generally happen,) or if the distance is called for at any point between flags, the fore-bearer must stop when his ring touches the flag-staff, or other point required, and cry *links*. Thereupon the hind-bearer, after carefully stretching back the chain, must count off the links back of the standing stake, and after deducting them from the whole hundred links, render to the surveyor

the number of tallies on his belt, stakes in his left hand, and odd links.

Sec. 78. *Offsetts.* After measuring 17 chains, the hind-bearer gave me notice, that the pond F, was impassable. Whereupon I went back to the starting place and directed the ax-man to set a stake precisely in line with the flag at the end of the 17th chain. Then I set the compass at that point, and turned the sights slowly until the needle passed ninety degrees; which brought them to the course S $46\frac{1}{4}$ E. I ordered the ax-man to stand in that line at *l*, a distance sufficient to clear the pond; which on measuring I found to be 8.50, where he set in a stake. There I set the compass and turned the sights, till the needle settled on the course of the line, N $43\frac{3}{4}$ E, and ordered the ax-man to stand at *i*, a distance sufficient to clear the pond in that direction, where he set in a stake. On chaining to *i* we found the distance to be 22.50. There I set the compass on the reversed course of the first offset; to wit, N $46\frac{1}{4}$ W. Then I ordered the ax-man in that direction so far as to be sure to go a little beyond the original line. In this direction we measured 8.50, a distance equal to the offset; which brought us upon the original line, where the ax-man set up a stake. These stakes were set up, for the purpose of correction if required; but they had no influence on the survey, as the line *l i* was treated as a mere continuation of the line 1, 2, as if it had been measured across the pond.

Sec. 79. *Minutes.* After reaching the flag at 2, I made the common minutes, which stood thus:

N $43\frac{3}{4}$ E 47.80

At 17. Griswold's Pond, where I made an offset at right angles, 8.50.

At 39.50, having cleared the pond, returned to the line; the parallel offset line being 22.50.

At the corner 2, I set the compass and directed it to the flag at the corner 3. Finding the forward sight nearer the south end of the needle than the north, I set S in the field book; and as the same sight was on the east side of the needle, I set E in my field book; and its distance east was $42\frac{3}{4}$ degrees. On chaining to the flag at 3, the distance in measure was 18.20; all of which I wrote down as before.

On setting the compass at the corner marked 3, and finding the forward sight nearer the north end of the needle than the south, when directed towards the flag-man at corner 4, I set N in my field book; and as the same sight was on the west side of the needle, I set W in my field book; and as its distance was 16 degrees, I wrote down N 16 W.

SEC. 80. *Heights and distances.* While on this line I took the necessary observations for ascertaining the distance and height of the meeting house A, as follows: At *o* I called on the hind-bearer for the distance run on this line; which he rendered 16.30. Here I directed the sight to the south-east corner of the house; and found its bearing to be N 65 W. Then I directed the cross levelling marks on the sight to bottom of the house, and found it to be on a level with this station. Ranging the forward sight until the tangent marks ranged with the top of the steeple, I found, by the tangent scale of Kendall, before described, the angle between the lines of direction to the base of the house and top of the steeple to be 4½ degrees. At *u* I called for the distance as before, which was rendered 32.40. Here I took the bearing of the same corner of the meeting house and found it to be S 51 W. These bearings, &c., I minuted for future calculations. No other extra minutes were made on the line, nor on the line 4 to 5, excepting the places of crossing Stoney Brook.

SEC. 81. *Random line.* On setting the compass at the corner marked 5, I could not see the corner marked 6; for an extensive piece of woodland was to be crossed. The corner boundaries were agreed upon by my employers and their neighbors; but the connecting line had never been run. Being obliged to run a random line, in order to obtain the true course, I set my employers and their neighbors to guess at the direction of the corner 6. Each stood at the corner 5 and pointed at a tree, which he supposed to be in the direction of the required corner. After taking the mean average of their opinions, the course was S 2 E. That course I pursued, by sending the flag-man along in that direction, as far as I could see him, then crying right, left, &c., until he came in the direction of sights; then crying *stand*, he waited for me and the bearers to come up—again went ahead, &c., until we came against the corner sought. Then I turned the sights through 90 degrees,

as when taking the offset at Griswold's pond. Calling on the hind-bearer he rendered 57.80 for the length of that line. On measuring the distance to the true corner, we found the error 3.10.

Sec. 82. *Extemporaneous calculations.* Not having a table of natural sines with me, and not being able to proceed until the course was corrected, I calculated the error in the course by using the formula 57, thus: as the distance run (57.80) is to 57, so is the error (3 chains and 10 links) to the error in degrees: or 57.80 : 57 : : 3.10. The answer was found to be 3 degrees; which added to the course S 2 E, gave the true course S 5 E. The true course being found, we returned to corner 5, and run the true line by the compass without the chain. The ax-man marked the trees which fell in the line, and the chain bearers were employed in throwing stones about the most important ones.

Sec. 83. *Ascent and descent.* We run the line from 6 to 7, as in other cases; and found it to be south 16.07. But in running the line 7 to 8 we crossed a steep hill, which required particular attention. For the other sides being either on level ground, or moderate slopes, this side would be disproportionably long, if the line was run without taking any notice of the ascent or descent of the hill. After running a random line (which was necessary on account of the interposition of the hill) and finding the course to be S 63½ W, we returned to the corner 7, and chained the true course, on account of taking the ascent and descent on the true line. This we conducted as follows.

Sec. 84. *Calculation of ascent and descent.* At the foot of the hill, the hind-bearer rendered 3.60 as the distance run. I sent the flag-man to the top of the hill, where he set up his flag and held his hat against the mark on his staff, 5 feet from the ground—being equal in height to my levelling mark on the sights, after setting my staff 6 inches into the ground, as usual. I took the ascent, which was 6½ degrees. Then running up the hill, the hind-bearer rendered 7.30 as the distance from corner 7. The flag-man then went down the hill, and set up his flag-staff and held his hat as before. I found the angle of descent to be 3 degrees and 30 minutes. On measuring to the flag, the hind-bearer rendered 14.20 as the distance from corner 7. We then proceeded to corner 8, and the distance rendered was 19.85.

Sec. 85. In order to reduce this line to a straight one, as if run through the base of the hill so as to set it down accurately in the field book, I made the following calculation. The angle of ascent and descent not being above 10 degrees, the formula 57 applied equally to the base and hypothenuse of a right-angled triangle. Therefore I said, as 57 : 3.70 (the ascending line) : : 6½ degrees : 0.42 (the height of the hill)—then as 6½ : 0.42 : : 57 degrees : 3.68 (the base of the hill as far as the end of the line of ascent.) Then I calculated the line of descent from its termination, backwards in the same way—the angle being 3½ degrees—the line of descent 6.90, and the perpendicular the same as in the ascending triangle, it stood 3½ : 0.42 : : 57 : 6.84, the bases added, 6.84+3.68=10.52, the length of the whole base of the hill. This deducted from the sum of the ascending and descending lines, the whole line stands thus : Ascent and descent 3.70+6.90=10.60—10.52=0.08. Deducting the 8 links (which is the difference between the base line through the hill and the line over the hill*) from the whole surveyed line, the true line for the field book was 19.77.

Sec. 86. *Common running.* We run from 8 to 9, which was west 17.02, and from 9 to 10, which was N 33½ E 19.00, without any important occurrence. Arriving at the corner 10, we proceeded, as to be related in the next section.

Sec. 87. *Oblique offset.* Setting up the compass at the corner marked 10, and directing the sight to the flag at corner 1, (the place of beginning,) I found the corner to be N 79 W. But the pond G obstructed our direct measurement. On running to the margin of the pond *r*, which was 3.30, I found the offsets could not be made at right angles, as had been done at the pond F, on account of the pond H. Therefore I directed the flag-man to go between the two ponds, until he should come to a place where he judged that a line parallel to the course of the line would clear the pond G, which

* I calculated this at full length, partly for the purpose of shewing the trifling difference in most cases of uneven surface, and partly to shew the accuracy of the rule, when the angle of ascent and descent does not exceed 10 degrees. The practice of directing the bearer who is at the lowest end of the chain to raise it to the horizontal level, is merely professional quackery. It is impossible that any thing like accuracy should be effected in that way. Neither could I rely upon this formula, if the angles exceeded 10 degrees. But I would take the angles in the field and calculate the error by plotting a profile of the ascending and descending lines, or with tables.

brought him to 6. On directing the sights towards the flag, I found the direction to be S 59 W, and the chain bearers found the distance 13.50. Then setting the compass at 6 with the sights on the course N 79 W, as before, and setting the flag-man at *t*, then measuring to the flag, we found this parallel line to be 15.70, which is to be added to the 3.30 measured from 10 to the pond. Now we return to the line at W, by measuring 13.50 on the offset course reversed—(that is, the offset course being S 59 W, the reversed course is N 59 E.) From W we continued on the course to 1. Adding the two measured parts of the true, and the parallel part, 6 *t*, of the offset, the distance, to be set down in the field book, was 32.60.

Sec. 88. *Survey closed.* Having completed the survey of the outbounds of the farm, we retired to the dwelling house to make the calculations, which were necessary to be made before the farm could be divided. We dismissed all the men employed, and directed them to return on the 2d of April. For we had occupied the 30th and 31st of March in the survey ; and it was a full day's work to make the calculations.

Sec. 89. *Field book.* The following is a copy of my field-book as made in the field. The original entries should always be prepared by the surveyor, and a fair copy made for the employers.

Field Book of an Agrometric Survey,

For William and Robert Gardner, commenced at 7 o'clock, A. M., March 30th, 1801.

Bearers, John Bemis and William Miller. *Ax-man,* William Babcock, jr. *Baggage-man,* William Clarke, jr. *Flag-man,* Levi Morris.*

Beginning boundary, stake and stones on east bank of Stoney Brook, N 10 W 13. from its junction with Meadow Brook, and S 23 W 11. from the S. E. corner of Griswold's mill.

1. N 43$\frac{3}{4}$ E. 47.80

* All assistants' names should be entered in the field book; because a reference to them may adjust differences of opinion, long after the death of the surveyor, and their names being found among his field notes would save much inquiry.

17.00 offset right, 8.50, at right angles.

39.50 offset left, 8.50, at right angles.

2. S $42\frac{3}{4}$ E. 18.20

3. N 16 W 45.20

16.30 meeting house bears N 65 W.

Same, angle of elevation to top of steeple $4\frac{1}{2}$ degrees to base level.

32.40 meeting house bears S 51 W.

37.10 Stoney Brook, 1.50 across.

4. S $77\frac{1}{4}$ E. 19.00

5.20 Stoney Brook, 1.10 across.

5. S 5 E. 57.80

0.00, run random line S 2 E 57.80—woods.

57.80, run left to the true corner 3.10

0.00 returned and run on true line, S 5 E.

6. South 16.07

7. S $63\frac{1}{2}$ W. 19.80

0.00 run random line S 60 W 19.80—hill.

19.80 run right to the true corner.

0.00 returned to beginning and run on true course, which had been found by calculation, S $63\frac{1}{2}$ W.

3.60 take ascent of hill, $6\frac{1}{2}$ degrees.

7.30 top of the hill—take descent of it, $3\frac{1}{2}$ degrees.

14.20 bottom of the hill. Calculate here and find base line 5 links shortest, which I deduct.

8. West 17.02

9. N $33\frac{1}{2}$ E 19.00

10. N 79 W. 32.60 to place of beginning.

3.50 Meadow Pond.

Same, offset S 59 W 13.50 left.

19.20 onset N 59 E 13.50 right.

Compassed the farm March 31st, at 6 o'clock P. M.

Sec. 90. *Map and casting.* As my employers were very desirous that the contents of their farm should be cast up with extreme accuracy, I cast it by the three best methods in use. 1. By *separate triangles.* 2. By reducing it to a *single triangle.* 3. By the *trapezoid* method, called the rectangular, or latitude and departure, method.

As a map is always required, the survey should be plotted before any calculations are made. Accordingly I prepared to plot the survey by ascertaining how it would lie upon a piece of paper of a size adapted to the object. To do this, I laid down the courses and distances on a slate, by guessing at the course and length of each. In this way, I found that I must begin on the left hand side of my paper, about two-thirds down towards the bottom. As the map was required to fit a common pocket-book, I assumed for a scale of equal parts, 20 chains to an inch. This is too small a scale for accurate calculation; therefore I drew another map with a scale of 4 chains to an inch. But I shall not exhibit that plot here; as I proceeded with the large map in all respects as with this.

Sec. 91. *Abstract and meridians.* I prepared the paper for plotting by drawing parallel meridian lines, by the opposite sides of the wooden inch ruler, before mentioned; as *i i*, *o o*, *u u*, &c., in the figure. Then I copied the courses and distances from my field book, and reduced the distances by dividing by 20, my assumed scale per inch, so that I could plot by an inch diagonal scale, thus:

1.	N $43\frac{3}{4}$ E	47.80=2.39
2.	S $42\frac{3}{4}$ E	18.20=0.91
3.	N 16 W	45.20=2.26
4.	S $77\frac{1}{4}$ E	19.00=0.95
5.	S 5 E	57.80=2.89
6.	South	16.07=0.80
7.	S $63\frac{1}{2}$ W	16.80=0.99
8.	West	17.02=0.07
9.	N $33\frac{1}{2}$ E	19.00=0.95
10.	N 79 W	32.60=1.63

Sec. 92. *Plotting.* Having drawn the meridian scratch-lines, I laid the centre of the protracter precisely at the starting point 1, with its straight edge on the line *i i*. Then I pricked the paper at the degrees $43\frac{3}{4}$ with a sharp needle, having its eye-end set into a handle. Taking 2.39 from the diagonal inch scale, in the dividers, I set this distance in the direction of the pricked point, guided by the scale-rule, and drew the line 1, 2. Next I laid the centre of the protracter at the point 2, with its straight edge as nearly parallel to the meridians as I could guess; and adjusted it *precisely* parallel

with the dividers, thus : I extended the dividers to the meridian line of the most convenient distance from the centre of the engraved line on the face of the bevel, and with the same extent of the dividers applied one foot to the engraved line near each end of the straight edge, and the other foot to the meridian line. Then I pricked the paper at the degree $42\frac{3}{4}$, and set the distance on the line 2, 3. Thus I proceeded with all the lines, until the plot closed at the starting point.

Sec. 93. *Proof of a survey.* If the plot had not closed, I should have measured across the farm, near the middle, to find where the error was probably committed. For example, if a line was run from corner 1, to corner 6, and should be found to agree with the same line drawn across the map, the error must have been committed after passing the corner 6. But if it should have been found too long or too short, or if the course of the line should come out wrong, the error must have been committed between corners 1 and 6 ; and the nature of the mistake could have been determined by the length and direction of the line. I should then have re-surveyed the sides among which the error was committed. If the sides of the farm are numerous, several such cross lines may be run, sub-dividing the erroneous part of the survey, until it may be driven to two or three sides, before the re-survey is made. But I would never let a survey go out of my hands, until it had closed accurately, with perfectly accurate instruments.

Sec. 94. *Triangular cuttings.* After I had plotted the farm with great care, and closed the plot, I proceeded to cut it into triangles. I kept these objects in view while cutting it up. 1. To avoid making acute angles when possible. 2. To make one base serve for two triangles, whenever it is practicable. 3. To make the outside lines the bases of as many triangles as possible. When any of these advantages interfered, I gave the preference according to circumstances. The triangles A and B might have one base, 2, 10. But I preferred taking the trouble of two multiplications, to save the outside line 1, 2, for a base. I saved an outside base in C and D. As it was hardly practicable in any other cases, I put E and F on the same base; also G and H.

Sec. 95. *Triangular castings.* Having measured all the bases, and all the perpendiculars, I proceeded to calculate the areas in

the usual way. That is, I multiplied the base and perpendicular together of each triangle separately, excepting where two triangles had the same base in common. In such cases I added the two perpendiculars and multiplied both by the base, at once. After adding all the products and halving the sum, I multiplied it by 400, the square root of the divisor, by which I reduced the scale, for the convenience of plotting by an inch scale. This is called "*raising the scale*," by old surveyors—or "*restoring true measure.*" Then I reduced the chains and links to acres, quarters, and rods, by continually dividing by 10 (the number of chains in an acre,) and continuing the same divisor through the reductions to quarters and rods.

The scale may be raised, before multiplying the bases and perpendiculars together, by multiplying each base, and each perpendicular, by the divisor; as by 20 in this case. The following castings are made upon that method.

CASTINGS.

		Bases.	Perp.	Products.	
A	1 to 2,	47.80×	27.70	=1324.06	2)4750.72
B	10 to 2,	41.80×	12.80	=535.04	10)237|5.36
C	3 to 4,	45.20×	16.80	=759.36	4
D	5 to 6,	57.80×	9.70	=560.66	
E, F	3 to 7,	36.50×	5.00, 21.50	=967.25	2|144
					40
G, H	2 to 7,	35.55×	13.00, 4.00	=604.39	5|760
					A. Q. R.
			Double areas	=4750.76	Ans. 237 2 5

Sec. 96. *Triangular casting reduced to a single triangle.* In order to prove the accuracy of my calculation, I reduced the whole plot to a single triangle; and then cast the contents by one multiplication. Before exhibiting that operation, I will explain the principle with a small figure of but 5 sides, A, B, C, D, E. This method depends on the principle referred to in the 2d article in the 33d section. Having lost my brass slip on the road, I made one of a piece of tin with a pair of coarse shears; and I will explain this instead of a better, such as I now have before me. I cut a slip of tin half

an inch wide and 9 inches long, perfectly straight. In the middle I drew a line lengthwise, perfectly straight and exactly parallel to its edges, as exhibited in the figure. The object of this slip was the same as the groove on the protractor; that is, to exhibit parallels without defacing maps by marks, and to guide a foot of the dividers more accurately than could be done by a scratch on paper.

The principle just referred to is here exhibited by the triangles D C B and D G B. For both stand on the same base D B, between the same parallel lines P, P, and 1, 1. Consequently contain equal areas. Therefore by extending the line A B to G, and drawing the line G D, there is just as much land added to the field by taking in the area at B G. But it is not necessary to draw the parallel line P P. For if the dividers are extended from the angle C to the central line on the slip 1, 1, and then carried with the same extension to the indefinitely extended line F G, and moved back and forward on said line, until it rests at a point (as G,) when it is found (by sweeping the other foot,) to be the nearest distance from the central line on the slip, that point will be in the parallel line P P, and at the angle of the new triangle.

It will be seen, that by this operation, the angle at C is extinguished; leaving but four angles in the field. The angle E may be extinguished in the same way; leaving the single triangle F G D. By the use of the slip, all embarrassing scratches and marks on the plot are avoided; and the base and perpendicular of a single triangle only, are to be measured and multiplied by each other. Half of that product is the area of the field.

Sec. 97. *Choosing base lines.* When a farm of many sides is to have its area calculated upon this plan, several triangles will grow out of each other in a manner more complicated. But if the slip is always laid so as to connect two corners, leaving one between, that one will be extinguished. One side must always be assumed and extended indefinitely for the base line, as F G; every new corner must be made on that line. The side chosen for the base line must not be chosen on account of its greater length; for it will make the angles on it too acute for accuracy. But it must be so chosen, as to leave most of the plot standing upon it in its longest direction. After reducing the angles on one side of the plot, until they extend along the base line so as to make very acute angles, commence on

the other side of the field. But always work to the same base line, and always begin with the angle nearest to it.

Side A B 20.00		*New triangle.*
B C 21.50		F G 49.70
C D 26.00		D G 42.00
D E 25.70		
E A 36.60		2)208.74000
		10)104.37000
Scale 20 per inch.		4
		1 7.48
		40
		29 920
	Acr. Q. Rd.	
	10 1 29	

Sec. 98. *Two base lines.* It will often happen, that the angles formed on the base line will be too acute, even after working on both sides of the field. In such cases extend one of the new sides indefinitely, which touches the base line, and work to that as to the first base line. Then, when all the angles are reduced to four, extinguish whichever of the angles may be most conveniently extinguished; without regard to any choice between the base line, whether the first or second one be finally retained.

Sec. 99. *Single triangle accurate.* The advantages of this method over that of casting the contents by separate triangles are manifest. Every step in the process is wrought by points, and one metallic line. Most errors in plotting arise while working to the scratch lines on paper. If the points are pricked in with sharp round instruments, and the paper is old and of a firm texture, we can work to such points with more accuracy than can be expected from the most skilful survey. And a line accurately engraved on copper, and above all on tempered steel, will scarcely admit of an error. Considerable practice is necessary in this case, as in all others.

Sec. 100. *Trapezoidal method.* The third method which I adopted for proving my calculation, was the *trapezoidal,* or latitude and de-

parture method. It is constructed upon the following plan. Let a meridian line be drawn on the east or west side of the plot, so as to touch its extreme side or corner, as the dotted line 4, 9, in the figure, which touches the extreme corner marked 1. Let two lines be drawn perpendicularly from this line, so as to touch the north and south extremities of the plot, as 4, 4, and 9, 9. Now calculate all the area included within the meridian line, the two perpendicular lines aforesaid, and the outside lines from the end of one of the perpendicular lines to the other; as 4, 5—5, 6—6, 7—7, 8—8, 9. That is, all the land, both inside of the plot and outside of it, between it and the meridian line 9, 4. Then cast all the said outside part, and subtract it from the sum of both inside and outside area; and the remainder will be the inside area, or true contents of the survey.

Sec. 101. *Accuracy of the trapezoidal method.* The advantages presented in this plan are manifest on inspecting the figure. It will be seen that the whole may be cut into trapezoidal figures. Or that the north and south sides of each will be parallels, standing perpendicularly on the west line. All the sides but one, of each figure, are meridians and parallels of latitude; consequently they may be calculated like latitude and departure in traverse sailing. Then their contents may be found by adding the departures bounding each trapezoid, multiplying them by their latitudinal distance from each other, and halving the product. For example, the line 5, 5, (51.48) is the north boundary, and 6, 6, (57.15) the south boundary, and the line 6, 5, (57.56) the latitudinal distance, of a trapezoid. Then $51.48+57.15\times 57.56 \div 2=3126.3714$.

Sec. 102. *In and out areas.* By a little attention to the figure it will appear, that when the measure of the latitude is from north to south, it gives the length of the trapezoids both in and out of the plot; and when the measure of the latitude is from south to north, it gives the length of the trapezoids outside of the plot. Hence it follows, that when the southern measure is the multiplier the products must be added together for the inside and outside area; and when the northern measure is the multiplier the products must be added together for the outside area, and subtracted from the other area. When an inner angle extends backwards, as that marked 3, the area

is cast in and out twice; but still the rule does not require any variation.

Sec. 103. *Form of arrangement.* To avoid confusion, the following tabular formula was constructed. It will be understood by inspection, after reading the preceding sections. Traverse tables, such as are used in navigation, are used for finding the latitude and departure in this kind of calculation. But it is almost as easy, and much more simple, to multiply the chains and links by the sine of the course for the latitude, and the co-sine of the course for the departure.

Sec. 104. *North and south areas.* In the figure the single dotted lines on the dotted side of the double lines, are the departures of northern areas. And the single lines of long dots and the long-dot sides of the double lines, are the departures of southern areas.

Courses and Distances	N.	S.	E.	W	1 dep	2 dep	N. Area	S Area
1. N 43 3-4ths E 47.80	34.53		33.05		33.05	33.05	1141.2165	
2. S 42 3 4ths E 18.20		13 37	12 35		45 40	78.45		1048.8765
3. N 16 W 45.20	43.45			12.45	32.95	78 35	3404.3075	
4. S 77 1 4th E 19 00		4.19	18 53		51 48	84 43		353.7617
5. S 5 E 57.80		57.56	5.67		57.15	108.63		6252 7428
6. South 16.07		16.07			57.15	114 30		1836 8010
7. S 63 1-2 W 19.80		8.84		18.71	38.44	95 59		845.0156
8 West 17.02				17.02	21 42	59 86		
9. N 33 1-2 E 19 00	15 84		10.49		31.91	53 33	844 7472	
10. N 79 W 32 60	6.21			31 91	00 00	31.91	198 1611	
	100.03	100.03	80.09	80.09			5588.4323	10337.1976
								5588.4323
								4748.7653

Or 237 Acr. 1 Qr. 30 Rds.

Sec. 105. *Merits of the three methods.* Having cast the contents of the farm by three methods, all of which I have long used in practice, it may be proper to express my opinion on their relative merits. I say, most decidedly, that the method of plotting and reducing the plot to a single triangle, is the best known method, for ordinary cases of farm surveying. For smooth even plains, and for city lots, the trapezoidal method is best. I have no room here for my reasons at length. But who will not perceive at a glance, that uneven land requires an averaging method, which is not practicable by any method but by accurate plotting? Moore himself (the inventor of this method, of whom I learned it personally before he published it,) acknowledged that defect in his *latitude* and *departure* method, as he named it.

Sec. 106. *Heights and distances.* Having completed the survey of the farm and cast its contents, it remained to calculate the distance and height of the meeting house, observed while running the line 3, to 4, before I proceeded to divide it. In the field book under section 89, it appeared that observations were taken at two stations, *o* and *u*, which were 16.10 apart. At one station the bearing being N 65 W, and at the other S 51 W, it appeared by inspection that the angle at *o* was 49 degrees, and at *u* 71. These subtracted from 180 left the angle at the meeting house 60 degrees. By section 44, the length of the line from the house to *o* was found thus: as .86603 (the sine of 60) : 16.10 : : .94552 (the sine of 71) : 17.57. Then I found the height of the meeting house thus: the angle of elevation being $4\frac{1}{2}$, I subtracted it from 90 degrees, and it left $85\frac{1}{2}$ degrees for the angle at the top of the steeple. Then as .99692 (the sine of $85\frac{1}{2}$ degrees) : 17.57 (the distance from the meeting house to *o*) : : .07846 (the sine of $4\frac{1}{2}$ degrees) : 1.38=91 feet. That is, the distance was 17.57, and the height 91 feet.

Sec. 107. *Heights and distances geometrically.* I made a calculation also, by a plot thus: I laid down the courses and distances, from the points *o* and *u*, indefinitely, and measured from *o* to the point of the intersection of the lines; which gave the distance. Then laying down that distance as the base, raising an indefinite perpendicular on one end, and laying down an indefinite line from the other end at an angle with it of $4\frac{1}{2}$ degrees, the point of intersection gave the height. But I laid down these plots by a much larger scale than I had used for plotting the farm.

Remark. If I had made frequent trials along the line 3, 4, so as to have found the point where the bearing of the meeting house would form a right angle with the line, both calculations might have been made by the *formula* 57, with and without reducing.

Sec. 108. *Division of land.* Before making calculations for dividing the farm, I inquired whether any points were fixed upon. I was thereupon directed so to divide the farm that William should have the north end, and 10 acres less than Robert; and that one end of the division line should be on the line 1, 2, at *z*, the margin of the pond, where we made the first offset. On looking over the map, [see section 81] I found that the corner 6, would be nearer to the termination of a division line, starting at *z*, than any other cor-

ner. I drew a line through z, 6, indefinitely, for a base upon each side of which a triangle was to be formed, as described in section ninety-six. By this operation I found 1287.50 on the north side, and 1086.88 on the south side, of said base line. Consequently the east end of the division line required to be moved 6.60; far enough to include half the difference, 100.31 added to half of the 10 acres=150.31.

SEC. 109. *Division by a triangle.* Here I was obliged to introduce a new application of the section 52, one, two, relating to the areas of parallelograms and triangles. For as a triangle is half a parallelogram, it is manifest, that if the quantity of land to be taken from the north side (5 acres and half the difference between sides) was doubled and that sum divided by the line z, 6, (found to be 45.50) the quotient would be the perpendicular of the triangle to be added; thus, $150.31 \times 2 \div 45.50 = 6.60$, the distance to which the east end of the line z, 6, was to be removed north. I drew the line z, a, and measured its course and distance on the map, which I found to be S 87¼ E 44.00. Then I went to the field, run and marked said line and set up the necessary boundaries. I gave each an entire field book of his separate share, after altering the line 6, 5, to accommodate the divisions to the divided field notes.

Remark 1. It must not be forgotten, nor overlooked, that after finding, that 300.62 was the difference between what was made by the assumed division line, z, 6, and what was required, but half that sum was to be taken from the north part—as one acre taken from the one part and added to the other, will make a difference of two acres between the parts.

Remark 2. This method of dividing a farm, and casting the parts separately is an excellent method for proving a survey. A farm of numerous sides may be thus divided into three or four parts for more perfect proof of accuracy. It may be farther observed, that if we were sure that the survey and calculations were correct, a farm might be divided by a calculation made on one side of the cross line only.

SEC. 110. ROAD SURVEYING. Road surveying, when nothing more is required than courses, distances, and notices of objects, belongs to Agrometry. But when ascents, descents, dug-ways to be made, beds to be raised, &c., are to be calculated, it belongs to

Engineering, described hereafter. In road surveying, however, the field notes are kept differently. The chain is carried directly on, without starting anew at the angles or turns in the road, as in farm surveying. But the change of course is always set down, like other incidents—and the leathern tallies are carried as directed in section 77, until the seven are moved, and then another tally is run; when the hind-bearer will be reminded that eight tallies are out, by finding no tally to slide. Then he notifies the surveyor that one mile is completed. This is entered in the field book, unless the hind-bearer is also entrusted with the mile entries.

Sec. 111. *Road field book.* The following is the form of the field book of a common road survey.

Field book of a Road Survey from Catskill Village to Catskill Mountains.

Beginning at the west end of Benton's bridge.
0.00 S 85 W.
10.21 Meiggs' house, left.
31.20 Gordon's house, right.
36.07 Cross road from Catskill to Fall mills.
42.90 S 72 W.
140.31 Long Swamp.
247.60 N 64 W.
317.90 S 86 W.
560.41 Kisketam flats, at Coat's land.
640.27 Greene Patent ledge in M. Lawrence's range.
Same N 79 W.
720.00 Main Catskill Mountain.

III. OROMETRY. (*Line surveying, reviving lost lines.*)

Sec. 112. *Running lines from a plot.* In *Orometry*, the courses and distances are always given; but the objects of the survey are various. Sometimes the out-bounds of large tracts of land are run and established, and the tracts are cut into lots on paper; then the business of the surveyor is to run, and mark, the boundaries. In such cases the surveyor must be very careful to plot from level out-boundaries; and, in cutting up the tract into lots, he must be

equally careful. For without such care, future tenants, or purchasers, will have good reason to complain of uncertain boundaries.

Sec. 113. *Reviving old lines.* But after a district of country is inhabited, the most common cases in Orometry are, the running and marking of old lines, lost by negligence. In such cases, if any one corner is remembered and can be precisely located, all the other corners and lines can be found. Much experience, however, is required for searching out old lines. If several boundaries accord with each other, this accordance has great weight with honest farmers, also with jurors, in fixing the lines. For no one will doubt the correctness of lines, when several sides of a farm coincide with the written courses and distances, even if no well established corner can be found.

History of an Orometric Survey.

Sec. 114. *Reviving lost corners.* On the 21st of June, 1803, I commenced the survey of the tract of land, called Dise's patent, in Schoharie county, New-York. It had been surveyed in the year 1743,* sixty years before I surveyed it. The boundaries were lost in most cases; and the proprietor of Scott's Patent (which belonged to John Livingston, Esq., whose agent I then was) accused Mr. Rechmeyer, the proprietor of Dise's patent, with encroaching upon his tract. A survey, therefore, became necessary. But there was not a corner boundary established. Several side lines were pretty well marked.

Sec. 115. *Settling magnetic variation.* The first point to be settled was, the variation of the needle from 1743 to 1803, a period of 60 years. Having been told by the Surveyor General, De Witt, by Messrs. Cockburn, Wigram, Trumpbour, Van Alen, and Savage, of the State of New York; also by Samuel Moore, of Salisbury, Connecticut, that the north point of the needle had been approaching the meridian for a century, at the rate of nearly one degree in twenty years, I adopted that fact for my rule of calculation.

Sec. 116. *Proving the variation.* In order to make the allowance, without the possibility of mistake, I concluded to run the long-

* I surveyed this tract at that time, (June 21st, 1803,) but it is too large to present the whole survey here. Therefore I leave out several sides, and I leave out some precise dates of the patent also.

est boundary line first, on which any marked trees could be found. Not being able to find a corner, I set down the compass at a place on the line where I found several marked trees nearly in a range apparently agreeing in direction with the given west line; which was S $2\frac{1}{2}$ E. I let the needle settle on the given course and then turned the north point east three degrees with my finger; as it had moved east three degrees in sixty years; according to the opinions of our most experienced observers. This brought the north point half a degree east of north; consequently the course to be run was S $\frac{1}{2}$ W. Then I turned the sights (after replacing the glass cover) so that the south point of the needle rested $\frac{1}{2}$ west. I run on that course by directing the flag-man as usual, until I became satisfied that this was probably the true line, from the number of marked trees which coincided with it.

Sec. 117. *Proving old marks.* Lest these should be spurious marks, I directed the ax-man to cut in above and below several of them until dead wood appeared; and then to split out blocks at the depth of the dead wood. Here we found, more or less distinct, J. D. (for John Dise) 1743. On the outside we found a distorted D on several trees. On counting the grains (concentric cylinders) we found from 40 to 50 very distinct, in several cases; and indistinct ones, which might supply the remainder of the 60 required to answer to the 60 years. As all the new layers of wood are introduced between the bark and wood in the form of mucilage (cambium) which hardens into thin concentric layers, all the grains appear as if no mark had ever been made on the tree; excepting those near the original interior mark, and the distorted outside mark.

Sec. 118. *Finding old corners.* As no corner could be found at any place which could be relied on, and as this line and the north line run through ancient forests where numerous marked trees remained, I concluded to run these lines until they should cross each other; and then to assume the crossing for a starting corner. By tracing all the lines from that corner, if I found a coincidence with similarly marked trees on several sides, I should believe I had run the old lines truly. On making the trial, I succeeded to the satisfaction of all parties concerned. But the allowance of one degree for 20 years was certainly too much by several minutes.

Remark. At the present time (1837) it is well known that the needle was stationary about the beginning of this century in this district; and that it is now on the retrograde.

Sec. 119. *Aberrations of needle often imaginary.* In Catskill, Greene county, New York, I run the boundary lines between two tracts called Row Patent and Greene Patent, in the spring of 1808. I could not close the Row Patent by four chains; though I plotted it several times with particular care. I concluded to resort to a survey, with a view to ascertain the side on which the error was committed. By comparing the descriptions, finding corners with cross lines, as in the Dise's Patent, I was able to settle with satisfactory proof, that the fore-bearer must have lost 4 stakes, which the hind-bearar omitted to correct. As the survey had been made 70 years before, none of the assistants could be found; though all their names were found in the field book of the surveyor, George Metcalf, Esq. While running one of these lines my needle varied; and I was obliged to send home for another compass which traversed well. It was at this time, I first observed, that my needle varied in that compass always when the sights were directed nearly as on that course. This led to the discovery, that most, if not all, the deviations of needles, which are ascribed to the attraction of iron mines, are caused by fine grains of iron, left in the card or limbs of the compass.*

IV. UDROMETRY. (*Marsh and aquatic surveying.*)

History of an Udrometric survey.

Sec. 120. *River and harbor survey.* I was employed to survey a trunk of Hudson river between Albany and Rensselaer counties, for the purpose of ascertaining which side of a middle ground ought to be selected for the purpose of improving the best channel. On the east side the channel was known to be the deepest; but the bottom was rocky and difficult to excavate. But the west side was loose gravel, easily excavated.

* See Silliman's Journal, vol. 12, p. 14, where I published several facts relating to that subject; and proposed that the needles should be very sharp and capped with brass or silver. By this means the steel point of a needle may be kept at a little distance from the limb, and defended from rust at the sharp point. It is found to be perfect in practice.

Sec. 121. *Staking out the ground.* First I caused my assistants to set stakes at all the turns on both shores of the river, and at all the turns on the middle ground. Then I took soundings along both channels. Wherever I measured the depth, I set up a stake by tying a stone to one end by a very short rope, for an anchor. On the other end I marked the depth, and tied colored rags to it, that I might readily see it from shore. The spots on the river-plot represent the stakes, and the figures their various depths in feet.

Sec. 122. *Base lines.* I run two base lines on shore, where I could find the best ground; to wit *p*, *s*, and *w*, *t*. On these lines I made marks, at *p*, *q*, *r*, *s*, *t*, *u*, *v*, and *w*. I chose these places so that I could see all the stakes, each at two stations; at both of which a compass was set.

Sec. 123. *Taking bearings.* Setting the compass at *p*, I directed the sights to 21, 5, *z*, 18, *e*, and 12; and I noted down each bearing, numbering them from north to south. Removing to *q*, I took the bearing of all the stakes which were nearest to this station. In this manner I continued until I had taken the bearings of all the stations on both sides of the river—each from two stations, by one or two compasses.

Sec. 124. *Points of intersection.* It is manifest, that (as the course of the base lines had been taken, and the distances from station to station measured,) the distance of the stakes from each other, and from the base lines, will be indicated by the intersection of lines drawn according to their bearings. And as numerous bearings were taken at each station, the plotting is easily performed; for all the bearings from one station may be marked off without taking up the protractor.

Sec. 125. *Lines of the shores.* By measuring at right angles from the base lines to the staked turns on the shore, or to any other accessible object, they may be laid down by erecting perpendiculars of the measured lengths, at the places noted on the base lines. These places may be connected so as to give the true form of the shore, &c.

Sec. 126. *Notations on the map.* The position of the stakes, with the depth of the water at each, was marked on the plot or map, at the intersections, as shown on the map. Notes relating to the

bottom, were made in the field notes, with references to the stake as marked down.

Remark. All harbor surveys may be made upon this general plan. Also any surveys of ponds, marshes, &c. But particular cases require a judicious plan, adapted to its peculiar circumstances; and no set of rules can apply in all cases. The surveyor's judgment will always give character to such surveys.

STATICS AND DYNAMICS;

As far as they are necessary to Civil Engineering.

SEC. 127. STATICS.* The science of gravitation or pressure, while bodies are restrained from motion; as the mechanical powers when in equilibrio, or the compression of metals or timber under heavy weights. The construction of bridges, piers, roofs, dams, flumes, &c., requires to be designed according to the laws of statics.

SEC. 128. DYNAMICS.† The science of motion, or moving force, applied or applicable to bodies when free to receive motion. As the power applied to a lever while raising a weight—the power of gravitation in giving motion to a carriage down an inclined plane—the power applied by means of a pulley in raising casks, &c.

When Statics and Dynamics are applied to water, they are denominated *Hydrostatics*‡ and *Hydrodynamics*.§

SEC. 129. HYDROSTATICS, applies to water, while its pressure is resisted; as the strength of planks and their fastenings on the frame-work of flumes, must be in proportion to the square root of the head of water.

SEC. 130. HYDRODYNAMICS, applies to water in motion; as the velocity of water in a raceway will be in proportion to the inclination of the plane of its descent. The various instruments and structures employed in hydrodynamics, are called Hydraulics.

* *Statos*, Greek, (from *istemi*,) standing, being stationary.

† *Dunamis*, Greek, power, efficiency.

‡ *Udor*, Greek, water, prefixed to *Statos*.

§ *Udor*, Greek, water, prefixed to *Dunamis*.

FALLING BODIES.

Sec. 131. All bodies, both solid and fluid, are accelerated equally by the attraction of gravitation. This is demonstrated by what is called the Droppers experiment with the air pump. The droppers receiver is a glass cylinder, usually about 18 inches in height. The air being exhausted, a feather and a piece of lead are dropped from top to bottom, by means of an appropriate apparatus. As the feather reaches the bottom as soon as the lead, it follows, that it is the atmospheric air only, that causes the difference of velocity in all common cases of falling bodies.

Sec. 132. After numerous and extremely exact trials, it is found that all bodies fall 16.2 feet in one second, in a vacuum. Also, of cou se, that in falling one foot, a body acquires such an increased velocity, that should its increase be suspended at the end of the foot, it would thence forward move at the rate of 8.1 feet per second. But the continued increase carries it 16.2 feet in one second from its starting. This principle will be more clearly illustrated with water-pressure.

WATER,

As an Agent in Engineering.

Sec. 133. Students should learn, from trial, to estimate water with facility, by weight, cubic measure, and common liquid measures. A pint of pure water weighs one pound.* The weights and measures adopted in this treatise, will be 2000 ℔s. neat weight for a ton, in accordance with the revised laws of New York. Also a cubic foot of water to 60 ℔s., and 28.8 cubic inches to a pound or pint of water. Let every student measure and weigh water by using common pails, cups, tubs, &c. Let vessels of all forms be measured by the inch as an exercise; and the correctness of the measures proved by weight and a sealed liquid measure.

* It is uncertain which was first adopted, the pint measure or pound weight; but it is evident that accident did not cause their agreement.

Sec. 134. *Hydrometers* should be immersed in water also; particularly *Baume's Areometer.*

Manner of using Baume's Glass Areometer* *in ascertaining the specific gravity of liquids.*

In constructing this instrument two stationary points are assumed; and if you have none at hand, these points may be found as follows. Take a slender glass tube, with a hollow bulb at the bottom. Put into the bulb mercury or fine shot, until you sink it in pure water almost to the top. Mark the zero point at the surface of the water. Then weigh 85 parts of water and 15 parts of table salt (muriate of soda.) After the salt is perfectly dissolved in the water, bring the temperature to 57° of Fah. Immerse the tube in this solution, and mark the point at the surface of the water, for the lower termination of 15 degrees. Being equally divided into 15 parts, these parts may be assumed as standard measures for any series of tubes (one ending where another begins,) for taking the relative specific gravities of liquids from the heaviest sulphuric acid to the lightest ether.

Water used in taking specific gravity of Solids.

Sec. 135. To be familiar with taking the *specific gravities* of materials for construction, is often of great use to persons in all other situations in life, as well as to engineers.

Illustration. Tie a strong silk thread or silk twine around a piece of marble weighing seven or eight pounds. Weigh it carefully, using balancing quarter-ounce or half quarter-ounce weights; so as to bring it to an even ounce weight on the steelyard bar. Then weigh it in water, sinking it so as to be wholly about half an inch below the surface of the water. Next subtract its weight in water from its weight in air—take this remainder for a divisor, and its weight in air for a dividend; and the quotient will be its true specific gravity. As the weight in air is 8 lb. $7\frac{1}{4}$ oz.; weight in water 5 lb. $\frac{1}{4}$ oz.

* *Araios*, Greek, slender or delicate; and *metron*, a measure.

In air,	8 ℔. $7\frac{1}{4}$ oz.	=	℔. 8.453
In water,	5 ℔. $\frac{1}{4}$ oz.	=	℔. 5.016
Rem.	3 ℔. 7 oz.	=	℔. 3.437

```
3.437)8.453(2.459
      6.874
      -----
      15790
      13748
      -----
       20420
       17185
       -----
        32350
        30933
```

spe. grav.; that is twice and about $\frac{46}{100}$ heavier than an equal bulk of water.

In this manner the solidity of materials for construction may be readily obtained—and it is preferable to the usual practice with grain-weights, for coarse materials.

Hydrostatics.

Sec. 136. Make a cylindrical bellows, by cutting two circles of thick board 10 inches in diameter, and nailing to the outside rim of each, with broad headed tacks, a hollow cylinder of leather. When finished it will present a leathern cylinder of strong calf skin, 10 inches in diameter and 8 inches long. Set in the middle of the top board a leaden tube of about the fourth of an inch in calibre, and 3 or 4 feet high. Let the top fit into a glass tube, 5 to 10 inches long, by a bandage of tow. When used, the leather needs to have been soaked in water several hours. Fill the cylinder with water through a plug-hole in the top board. Lay a weight on the top board, or let a student of suitable size stand on it, so that the water may rise into the glass tube. On measuring the height to which the water rises in the glass tube from the top board, and making the proper calculation, this result will be found: the weight set on will precisely equal the weight of a cylinder of water, 10 inches in diameter, of the height of the water in the tube. Hence it follows, that water presses according to its height; not according to its quantity by measure or weight. Therefore were it not for the impossibility of

maintaining the perpetual supply of water, a tube of an inch calibre would be sufficient for moving the machinery of an extensive factory, under a hundred feet head, supplied from a small reservoir or tub.

Hydrodynamics.

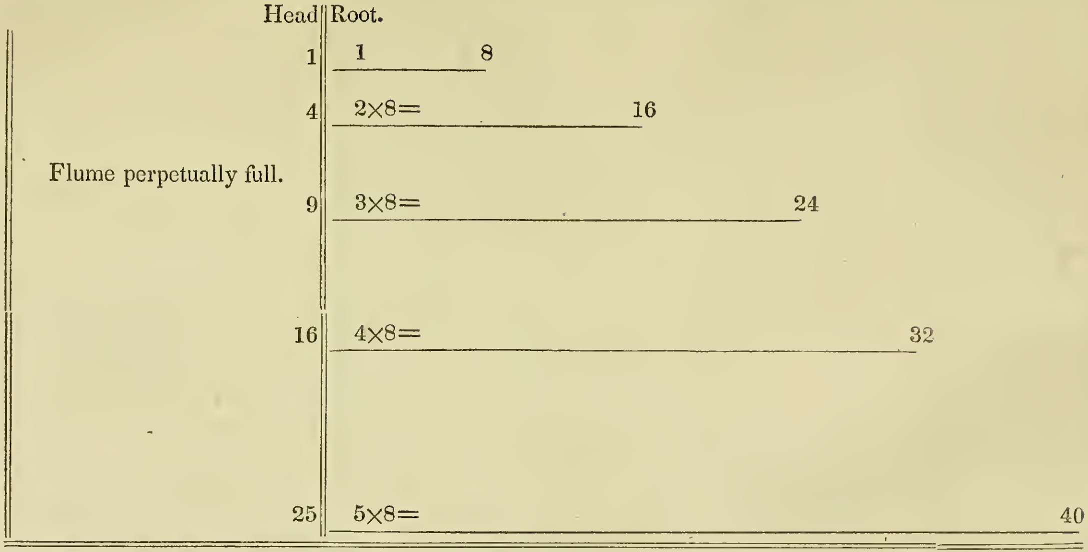
Sec. 137. After this is demonstrated, that water pressure depends on its height, and the weight of a standard measure of water is ascertained, we must determine by trial, what measured velocity will be given to water by a measured head. It was before stated, that trial has shown that a measured pint of pure water weighs a pound.

Sec. 138. Trial has also shown, that under one foot pressure, water will be forced through a lateral aperture with a velocity of 8 feet and a tenth, per second, in a vacuum—probably it will be correct in practice to say 8 feet per second in the atmosphere, at the precise point of effusion. This trial prepares us for the universal rule which governs in all cases of motion by gravitation.

Sec. 139. *The increased velocity of water effused, and of falling solids, is as the square root of the head of water, and as the square root of the distance through which solids fall.* Taking 8 feet of lateral effusion per second for the first foot, and the increase from that *zero* (if it may be so called,) as the increase of the square root of the head, and the increase of the distance fallen through in the case of solids, we arrive at results of vast importance in engineering.

Sec. 140. *Illustration.* A spacious flume has 25 feet of water in depth, with five apertures or gate-holes, of one square foot each. The centre of the first gate-hole is one foot below the top of the water—the second 4 feet—the third 9 feet—the fourth 16 feet—the fifth 25 feet. The square root of one is one, and the lateral effusion of water is 8 feet per second, as demonstrated by trial. The square root of 4 is 2, and the lateral effusion of water is $8\times2=16$. The square root of 9 is 3, and the lateral effusion of water is $8\times3=24$. The square root of 16 is 4, and the lateral effusion of water $8\times4=32$. The square root of 25 is 5, and the lateral effusion of water is $8\times5=40$. All intermediate heights of head may be calculated in the same manner. That is, extract the root of the height given, in feet and decimals of feet. Multiply that root by 8, the velocity of the first second of pressure. See the annexed diagram.

Effusion per second, as the square root of the head.



Scale, 10 feet per inch. Velocity as here exhibited at the instant of effusion; but gravitation gives curvilinear form to the stream effused.

Remark. Though the distinction between *Statics* and *Dynamics* is truly philosophical, it is inconvenient in its application to the Mathematical Arts—more especially so in a concise plain treatise, wholly devoted to practice. Olmsted's Compendium of Natural Philosophy is particularly recommended to students in Engineering, who have time to discipline their minds for a more systematic view of Mechanical Science.

Water under the influence of Atmospheric pressure.

Sec. 141. The atmosphere presses upon all bodies on the earth, at tide-water level, at an average of about 15 ℔s. on every square foot. Water is held down with a force, very unexpected by the student, until he makes the common trial, as follows: Boil water in the open air; and introduce the bulb of a thermometer at the moment of boiling. The mercury will rise to about 212° of Fah. Take off the atmospheric pressure *perfectly*, and it will boil at 67° As such accurate apparatus as this experiment requires is not common, an approximation to it must be used, which will satisfy every student. Put a gill of water in an oil-flask (I mean those Florence flasks, with a flag covering woven on them.) Cork it perfectly tight; and let one inch of the top of the neck be well wound with waxed thread, to prevent splitting by suddenly forcing in the cork. Before the cork is put in, boil the water for about one minute. This will force out the air, mostly. Thrust in a compact, soft, velvet cork (as the best are called,) while the water is boiling. Now let the water cool down to blood-heat; which may be known by applying the hand of a healthy person. If he can scarcely perceive any warmth to the hand on applying it to the bottom of the flask, the temperature is about 98 degrees, that is, 66 degrees above freezing. This is 114 degrees below boiling. As it is but 180 degrees from freezing to boiling, and as 114 is 66 above freezing; even this imperfect experiment proves, that almost half of the heat required for boiling water, is applied in resisting atmospheric pressure. And accurate experiments prove that more than two thirds of the heat is employed in counteracting the pressure of the atmosphere.

Sec. 142. Atmospheric pressure is most perfectly exhibited, by a tin tube about 34 feet in length. Let this be closed at one end,

and terminated by a few inches of glass tube. Fill it with water, and invert it in a vessel of water. The weight of the atmosphere will press upon the surface of the water in the vessel every where alike; but finding no resistance in the tube, (the air having been displaced by filling with water before inverting it,) the water is raised between 30 and 34 feet, until it counterbalances the weight of the atmosphere. A cheaper and more convenient method is, to use a torricellian tube, or (which is the same thing) a barometer. The height to which mercury is raised in the tube by atmospheric pressure, may be readily applied to water by calculation; reckoning mercury as $13\frac{1}{2}$ times as heavy as water. If the mercury rises 29.92 inches, multiply this by 13.5; it gives 403.92 inches—divided by 12, it gives 33.74 feet. This calculation is important in fixing the upper valve of a pump, in regulating water works, &c.

Sec. 143. *Illustration.* A pump maker had practised setting up pumps near the tide-water level on Hudson river. He was employed by a potash manufacturer in Castleton, Vermont, to erect a pump. He adopted his accustomed rule for fixing the upper valve. On a warm damp misty day (soon after the pump was set up) it was found that water could not be raised. The lecturer to the Medical class on Natural Philosophy, was called on by the proprietor (who was a trustee of the institution,) for an explanation of this strange phenomenon. It was found, that when the air was exceedingly light, by warmth and by being surcharged with vapor, mercury would not rise in the torricellian tube to a sufficient height to carry, by calculation, the water to a sufficient distance above the upper valve to give play to the piston. Lengthening the piston-rod and lowering the upper valve, immediately corrected the embarrassment.

Sec. 144. *Illustration.* I have been told that the head of the water works at Hudson, N. York, (two miles out of town,) was more than 30 feet lower than a high ground, over which the pipes were laid. I have often seen the head of the works in the present century, which were in good order; but this is said of their commencement some 40 or 50 years ago. True or false, the principle may be illustrated by supposing it true. It is said, that in damp warm weather, the water would not run. This is in accordance with the laws of pressure. Atmospheric pressure will carry water, in close

air-tight pipes, over hills from 30 to 34 feet above the spring head, near tide-water level, as may be demonstrated to students, by the barometer or torricellian tube before described.

GONATOUS FORCES,

APPLIED TO BRIDGES AND OTHER FRAME WORK, AND TO STONE AND BRICK ARCHES.

Sec. 145. *Gonatous forces* (from *gonu*, *gonatos*, Greek, angular flexions, like the knee joint,) are applied by means of angular flexions of ropes or bars; as the genicular braces of carriage calash-tops, or the sailor's funicular advantage in hoisting a sail by springing the halliards out from the mast. The laws of pressure applied to bridges, arches, &c., are better explained on the *gonatous principle* than by any other means, as follows: The weight of a bridge may be made to rest on a point; and that point to rest on a single pillar of stone or wood. The pressure would then be downwards, and single in direction. A ton (2000 ℔.) would press directly on the earth at the foot of the post. But if it pressed on the meeting point of a pair of rafters, here would be a *resolution of force* into two equal parts; one half in the direction of each rafter. And the pressure being in the line of the woody fibre of the rafters, it would not crush them without very great weight. This pressure would not be directed downwards, as if on a pillar; but would press outwards as well as downwards, and tend to spread or separate whatever supported the rafters at the foot. If the angle at the meeting of the rafters should be 126° 52′, the spreading pressure would be double the downward pressure, according to the law of diagonal lines. For each rafter would be the diagonal of a parallelogram, twice as long horizontally, as deep vertically. This is the common method of estimating roofs, bridges, &c., also arches, by estimating short chord lines, separately taken. But the same results may be produced by the application of the gonatus principle, called the genicular and funicular power; and it is more simple and of extensive generality. It will be understood by a reference to the knee braces of chaise tops, or better by inspecting the printing press, to which this power is very advantageously applied.

Sec. 146. As the *genicular** and *funicular*† powers are very important in their practical application, let students make the following trials. Fasten two pullies against a wall about 8 feet apart. (If you have no pullies, smooth wooden pins, half an inch in diameter and five or six inches long, may be substituted.) Draw a very flexible cord or rope over them, and attach weights to their pendant ends. Let the pendant ends hang down from each pin about 4 feet. Now apply different weights to the middle, and to the ends of the rope. . By thus flexing the rope in the middle, this principle will be demonstrated. As half the weight applied to the middle of the rope is to one of the end weights, so will be the descent of the middle of the rope, to the length of the section of it between the middle weight and pulley. It will be perceived, that this is the law of the inclined plane, inverted.

Sec. 147. Students can easily be made to realize the almost universal application of the law to be deduced from this experiment. For though the flexion of the rope is downward, the principle is the same. At the commencement of the flexion of the rope, one pound will raise hundreds. But let the flexion continue to be increased, until the angle is reduced to 90°, and the farther flexion will require great weight. So if the rope was changed to a jointed flexed bar of iron, and that turned with the angle upward, it would resist great weight, if placed upon the outer point of the angle. The law of the inclined plane would perpetually apply. That is, the weight would press in a perpendicular direction proportioned to its pressure in a lateral direction, as the distance of the angle from a straight line with the ends of the bar, to the length of a side of it.

Sec. 148. The application of this law to frame-work or to stone arches, may be farther illustrated by setting the pullies or pins in the wall or ceiling, so that one shall be 4, 5, or 6 feet higher than the other. Then fasten the middle weight to the middle of the rope by a piece of twine. Here all the weights will hang down, presenting a fair exhibition of the action of the law of gravitation. And the same law will apply to the flexion from a straight line compared with the length of the line between the middle and the pulley. The student should be taught to plot arches in short chord lines; and

* *Genicular*, from *genu*, Latin, a knee.
† *Funicular*, from *funis*, a rope.

then to lay down each according to the laws of the funicular power; transferred to the genicular, as above described.

SEC. 149. An exhibition of the application of the genicular power to arches, &c., may be cheaply made, as follows: Get out about 12 strips of wood, like common rulers, about an inch and a quarter in width, one third of an inch in thickness, and a foot in length. Let these be united in parallel pairs, and the six pairs be joined by a two inch piece, forming a free double joint with screw rivets. This six feet of six free joints, may be bent into every form of arch; and, by tightening the joints with the screws, the arches may be made to exhibit all the various effects of pressure. By hanging common cast-iron weights on various parts of the various arches into which the joints may be bent, every view may be reduced to mathematical calculations.

MECHANICAL POWERS.

SEC. 150. The mechanical powers are, elementarily, but two—the *Lever* and the *Inclined Plane.*

LEVER is subdivided into *Lever proper*, *Wheel* and *Axle*, and Pully. INCLINED PLANE is subdivided into *Inclined Plane proper*, *Wedge*, and Screw.

SEC. 151. LEVER is *prying*, when the fulcrum is between the power and weight.

LEVER is *lifing*, when the weight is between the power and fulcrum.

LEVER is *radial*, when the power is between the weight and fulcrum.

WHEEL and AXLE is a perpetual lever, either radial or prying.

PULLY is a perpetual lifting lever.

In all levers the power is to the weight inversely as the side to which it is applied is to the side applied to the weight—the bar, or its equivalent, being balanced.

SEC. 152. INCLINED PLANE PROPER. In all inclined planes the power is to the weight as the height of the elevated end is to the length of the plane.

WEDGE. The power is to the resistance as the thickness of the head, to the sum of the length of the two slant sides. But the usefulness of the wedge does not depend much on its advantages as a

power, when applied to splitting rails, &c. But it gives direction to a very great degree of momentum, acquired by swinging a beetle, sledge, &c., and suddenly expending all of it on the head of the wedge.

Screw. The power is to the resistance as the distance between the threads of the screw is to the length of a circular thread. The screw is chiefly useful in giving a very advantageous direction to the lever, as in case of the cider-press. It is also useful in making delicate mathematical instruments; as an index on its head, or handle, may indicate the smallest possible degree of movement.

Remark. These general principles are sufficient; for students must see the instruments and use them. It is impossible rightly to understand them from mere description. Olmsted's Compendium of Natural Philosophy may be profitably consulted on the subject of these powers.

ARCHITECTURE.

Sec. 153. Though Architecture is made up of artificial materials, it is truly a science. The historical origin of some of the elementary principles is rather obscure; but in most cases their history is known. The early ornamental buildings were of stone and brick. And when wooden buildings came into use, the wood was made to imitate stone in general form, sculpture, and painting. Hence it was that pillars became the elements of Architecture, whether of stone, brick, or wood. It has been suggested that the Gothic style grew out of the ancient practice among the Goths, Vandals, and other northern barbarians, of binding together the tops of slender trees, for covering with skins, bark, &c. That this gave rise to the sharp arch, and other peculiarities pertaining to that order.

Sec. 154. The pillar consists of the *base*, *shaft*, *capital*, *architrave*, *frieze*, and *cornice*. A column includes the base, shaft and capital. The pillar is extended laterally, or rather spread out, so as to constitute walls, bases, capitals, cornices, &c. And when the form of a column is assumed, its proportions are extended to all parts of a regularly constructed building. And this principle is rigidly applied to bridges, ornamented boats, carriages, rail-road cars, &c.

Sec. 155. As pillars are the elements of the science of Architecture, they are to be first presented to the student. Figures or draw-

ings of the elementary pillars will greatly aid the student; therefore he is referred to the various treatises on that subject. Benjamin's Practical House Carpenter is the most popular work in this country; and, perhaps, quite as useful to the student as any other work. But the most efficient method of giving instruction on this subject is, to take students about a city or village; and first, point out the various orders of architecture by correctly constructed pillars—second, point out the errors; which are always abundant—third, point out the lateral extension of pillars and parts of pillars, in the construction of walls, ceiling, door casings, window casings, &c. The teacher should call his pupil's attention most particularly to bridges, and other works, which do not come particularly under the daily operations of common carpenters.

THE FOUR PILLARS.

Sec. 156. The four pillars are called *Tuscan, Doric, Ionic,* and *Corinthian.* The *Composite* order is of modern application. It is a fanciful intermixture of the elementary characteristics of some of the four orders—generally of the Ionic and Corinthian. This treatise being a mere practical text-book, will include the essentials of students' recitations, only.

TUSCAN ORDER.

Sec. 157. This order is the plainest and stoutest of all the orders. But it varies in proportion to the weight it is to sustain. In some of the cases of best appearance, the column is in height equal to seven times its diameter adjoining the base; and the entablature is two diameters. Its parts may be thus enumerated: a square plinth below the base; a base of a large moulding of a half cylinder; a terete, or tapering cylindrical column; a plain capital at the top of the column; a plain architrave sitting somewhat towards the inside of the capital; a frieze on the architrave; and a broad over-laid cornice, projecting considerably forward—the entablature is thus made very plain.

DORIC ORDER.

Sec. 158. This is also a plain order, and resembles the Tuscan.

A little more slender than the Tuscan, and more ornamented. The columns are often fluted; and the entablature and cornice are often ornamented with triglyphs and modillions.

IONIC ORDER.

Sec. 159. This order is always distinguished by the *volute* or scroll. It is more slender than the Doric order, and is often highly ornamented with various kinds of sculpture. This order is more adopted in this country than any other, in all buildings of taste. The columns are mostly fluted; and the architrave and frieze are more or less ornamented. The base is surrounded with more complicated mouldings than the Tuscan or Doric; and the modillions are larger and generally of good workmanship.

CORINTHIAN ORDER.

Sec. 160. This order is distinguished by its plumose capital, and its more slender and delicate proportions. The base generally resembles that of the Ionic, and its column is generally fluted. Its architrave, freize, and cornice, are ornamented with sculptured work and curvelinear modillions. But its high elegant capital gives it a degree of grace and beauty, far exceeding the other orders.

COMPOSITE ORDER.

Sec. 161. This order, though compounded of two others, has its true characteristics. It has the plumose capital of the Corinthian order below, and the volute of the Ionic order above—but the volute is generally elliptical in a vertical direction. Its architrave and frieze are often highly ornamented.

PEDESTALS.

Sec. 162. These are proportioned to their respective orders. They consist of *base, die* (parallelopiped trunk,) and a *cornice*—altogether about a third as long as the column supported by them.

PILASTER.

Sec. 163. Pilaster is a pillar of any of the orders, which has the appearance of being partly sunken into the wall—or it may be de-

fined, as a pillar with a thin tapering parallelopiped column. It consists of a plank, or slab of free-stone, attached to a wall, fire-jambs, &c., constructed upon the principle of some of the orders.

MISCELLANEOUS STRUCTURES.

Sec. 164. *Colonade*, a series of columns, which are united by entabliture at their tops.

Arcade, several receding arches in succession, penetrating into, or through a building.

Balustrade, a series of small pillars, as those supporting stair-railings, &c. A kind of parapet.

Attic, often used for a garret; because the pedestal-like pillars, which hide the roof (garret room) are called attics.

Parapet or *Battlement*, any low wall or balustrade, surrounding a roof or covering of any works; intended to conceal an unseemly part, or to defend it. Sometimes it surrounds roofs for the purpose of supporting the hand-railing of a walk.

Pediment, generally a small triangular front roof, set in upon the slope side of a larger roof.

Balcony, open gallery; as the iron galleries around upper windows, and highly elevated terraces.

Belvidere, (beautiful view. French.) observatory, and turret.

Cupola, a dome, or smallish room, on the top of a building; as a belfry, or hemispheric sky-light.

Terrace, elevated walk.

Coping, top or binding-stone, or binding timber, on a wall.

Saloon, a vaulted spacious hall.

Corridor, a large hall or passage to distant apartments. Sometimes applied to galleries or covered ways, leading around buildings.

Lintel, and *Threshold*, top and bottom pieces of a door opening—sometimes applied to the top, or covered part, of a projecting outside room.

Niche, applied to recesses in walls, for setting in urns, &c.

RAIL-ROADS, &c.

Surveying a Route for a Rail-Road, M'Adam Road, Turnpike Road, Canal, &c.

EXTEMPORANEOUS TRAVERSE.

SEC. 165. If a road of any kind, or a canal, is proposed, across a mountainous, hilly, or even a moderately uneven country, a kind of extemporaneous traverse should first be taken. This will give the directors such a general profile, or rather view of the country, at a small expense, as may enable them to judge with considerable accuracy, respecting the most expedient route for a *preliminary* survey. It may be conducted advantageously with a compass, chain, and barometer.

SEC. 166. Let the barometer be set up at the beginning of the line. Let the flag remain with the barometer, while the compass and chain are carried to the first considerable elevation or depression, within view of the flag. Here run a line, of a suitable length, between two stakes, for the base line of two triangles—the apex of the first triangle to be at the first station of the barometer, and the apex of the other triangle to be at the next station of the barometer. The base line being carefully measured, and the angles at each end, formed by the line with the bearings of the barometer at each station, will be all that is necessary for taking two strides with sufficient accuracy. As the barometer will give a good approximation to the true height at every station, and as the distances may be pretty accurately taken; a profile across an extensive district may be taken in a short time. From five to ten miles per day may be taken by an experienced surveyor, and a skilful barometer-bearer. If notes are extensively taken, much of the character of the country may be presented on, or accompanying. the profile and sketch-book.

SEC. 167. The barometer should be applied twice at every station in the same day. It is found, that when the atmosphere is uniform, and the barometer is not influenced by change of temperature nor moisture, the barometer will sink the lowest at about 4 o'clock in both forenoon and afternoon; and will rise the highest at 9 in the forenoon and 11 in the evening. This is supposed to be caused by an uniformly operating cause, above clouds or other modifications of aqueous vapor. Therefore the barometer ought to be twice set on

the same day and place ; so that where it stands at 4 o'clock in the forenoon or afternoon, it should again stand at 9 in the forenoon or 11 in the evening, and an average be made. But when this cannot be conveniently done, it may in some measure be approached, by assuming opposite times nearly in contrast; as 9 A. M. and 4 P. M. But the medium hours do not need contrasting; as 7 A. M. or 11 P. M. But all cases require, that morning vapors should be exhaled before the barometer is used; and that the season of the year should be chosen, when the weather is most dry and settled.

Sec. 168. *Formula for calculating Heights by the Barometer, according to Hutton.*

As the density of the atmosphere, consequently its weight, diminishes in a geometrical ratio of its height, and as logarithms of numbers are constructed upon the same principle, Hutton sought, with success, a formula for applying logarithms for taking heights of hills, mountains, &c., with the barometer. He found that, if the temperature of the atmosphere stood at 31 degrees of Fahrenheit, the difference in the first four figures of the logarithm, for every hundredth of an inch on the barometer between the bottom and top of a hill, gave just one fathom (6 feet.) Hence his rule: *If the mercury in Fahrenheit's thermometer* (always attached to the barometer) *stands at* 31°, take the height of the mercury in the barometer, in inches and hundredths of inches, at the top and bottom of the hill. Find the logarithm of both. Subtract the logarithm of the top from the logarithm of the bottom of the hill. The four first places of figures in the remainder are fathoms, and the remaining ones are decimals of fathoms. If the answer is required in feet, multiply the fathoms and decimals by 6, and the product will be the answer in feet and decimals of feet.

Sec. 169. If the thermometer stands above 31°, an addition will be required; to be produced by the following formula: Divide the fathoms and decimals of fathoms, by the constant number, 435; and multiply the quotient by the difference between 31° and the number of degrees on the thermometer. This product is to be added if the temperature is above 31°, but to be subtracted if below.

Note. If there is any difference between the temperature at the top and bottom of the hill, the average is to be taken.

A portion of a table of logarithms, sufficient for our highest mountains, is inserted at the end of this treatise.

SEC. 170. Gregory's formula has the advantage of being convenient for the memory in the absence of tables. A perpetual formula of 55 with three ciphers—55 degrees of temperature as the standard—44 with a cipher, for a corrective for deviations from said standard. His rule is: *Divide the difference between the top and bottom hundredths of an inch, by the sum of both heights; then multiply the quotient by* 55000, *and the product will be the height in feet.* But if the temperature differs from 55°, add the 440th part of the height for every degree it exceeds 55°; and subtract the same for every degree below 55°. Example:

Barometer.		*Thermometer.*
Top of hill,	30.00	Average,
Bottom,	29.80	68°—55°=13
Difference,	.20	
Sum,	59.80	59.80).200000(.00334

.00334		17940
55000 formula.		20600
1670000		17940
1670		26600
440)183.70000(.04175		23920
1760	13	2680
770	.12525	
440	.4175	
3300	.54275	
3080	183.70000	
2200	184.24+Answer.	
2200		

Latitude and Longitude Surveys.

Sec. 171. Very extensive *Rail-roads* (like that now in progress from Lake Erie to New-York,) or *Canals* (as that from Lake Erie to Hudson river,) should have all remarkable points, along the various proposed routes, accurately settled by their latitude and longitude. This would greatly aid the judgement of directors; and greatly benefit large districts of country, by furnishing established points for future reference.

Sec. 172. Latitude is most conveniently taken by the sextant, at noon. But Longitude ought to be taken, inland, in most cases, by the eclipses of Jupiter's satellites. I will, however, describe the methods of taking longitude by *Jupiter's eclipses* and by *three-hour lunar observations.*

Sec. 173. Taking latitude at noon with the sextant, requires a nautical almanac; though some of the larger almanacs of the common kind, contain the sun's declination, and may be used as a substitute. But no engineer should fail to provide himself with the Annual Nautical Almanac; always published three years ahead, by the Messrs. Blunts, of New-York.

Note. Mr. Gates, of Troy, will furnish them to order.

Sec. 174. In taking the *latitude* at noon, a reflector is necessary. Bowditch prefers a bowl of molasses to a glass reflector. Set a bowl of molasses (a large soup-plate is preferable,) on the ground, and take the angle between the sun and its reflected image in the molasses, by bringing them centre to centre. This gives the double altitude; as the distance of the sun's image, below the level of the surface of the molasses, is equal to that of the real sun above it. Halve this double altitude, which gives the true altitude.

Sec. 175. Having obtained the sun's true altitude, proceed to calculate the latitude, as follows: Look out the sun's declination—If the declination is north, (as it must be, from the 21st of March to the 21st of September,) subtract it from the altitude; and then subtract that remainder from 90 degrees, which will leave the latitude. If the declination is south, (as it must be, from the 21st of September to the 21st of March,) add it to the altitude, and subtract the sum from 90 degrees, which will leave the latitude. In short days, when

the sun runs low, an allowance may be made for refraction, according to the table of refraction at the end of this treatise.

Note. In the longest days of summer, the sun will be too high at noon to admit of double altitude within the range of the sextant. Several methods are in use for obviating this difficulty. The following method may be adopted: 1st. Take the double altitude of the ridge of a house-roof, or some other straight horizontal line. Then wait for noon, and take the single altitude of the sun above said ridge, &c., and add it to half the double altitude of said ridge.

Sec. 176. In taking the longitude by the *eclipses* of *Jupiter's satellites*, no instruments are necessary but a telescope and a good time-piece. On land this method of taking longitude is the best. Proceed as follows: Look in the Nautical Almanac, in the monthly table of Jupiter's satellites, and find the time of the nearest *immersion* or *emersion* eclipse. Be prepared with the true time and telescope. Direct the telescope to Jupiter, with the slide drawn so as to give its largest size, a few minutes before the time of the eclipse. With the eye on Jupiter, move the slide so as to diminish it, until the satellites come within the field of vision. Then wait until the immersion or emersion occurs. If immersion is to occur, expect its disappearance a little before an apparent contact. Both immersion and emersion will appear suddenly. Note the instant of its occurrence, by the watch. Then calculate the difference in time between its occurence and the time set in the Nautical Almanac. Allow 15 degrees of longitude for every hour, and the same proportion for minutes and seconds, and you have the degrees and minutes of longitude from Greenwich at London.

Sec. 177. In taking the longitude by *lunar observations*, a good sextant, or reflecting quadrant, and a good time-piece, are necessary. Look into the Nautical Almanack, and find the angular distance between the *moon* and the *sun* or a *planet*, or one of the nine fixed stars, which are used for this purpose, which may be seen at the time of night or day required. These stars have been selected so as best to accommodate every part of the earth, and to be of sufficient magnitude for observation. They are called *a* (*alpha*) of *Arietes*—*a* (*alpha*) of *Aldebaran*—*Pollux*—*Regulus* of first and second magnitude—*Spica*, first magnitude—*Antares*—*Aquilæ* remarkably bright—*Formalhaut*, small—*Pegasus*, *a* and *b*. By ex-

amining these stars on a celestial globe, or map, particularly Burrit's Atlas of the Heavens, the student may soon make himself sufficiently familiar with their relative positions, to find them at one glance of the eye. In taking lunar observations, half an hour of shewing is better than ten days of reading. The sextant must be set according to the Nautical Almanac, for the nearest third hour. As the time approaches, look at the moon and sweep for a star; but look at the star and sweep for the moon as they approach each other. Note the instant they touch, according to the almanac and watch. Then calculate the longitude by allowing 15 degrees for every hour's difference between your time and the time given for Greenwich at London. Students who have no experienced teacher near, must read the directions given by Bowditch.

Hours of the day are reckoned from noon to noon; counting from noon, to 23 o'clock and 59 minutes. Parallax and refraction must be allowed according to the tables at the end of this treatise.

Sec. 178. Should a surveyor be called to lay off a piece of ground of great extent, (as the Oblong, taken from Connecticut last century and joined to New-York,) which was to be a true north and south parallelogram, he would be under the necessity of calculating the breadth of a degree of longitude at the north and south ends of the tract, and projecting it upon Mercator's method. To find the breadth of a degree of longitude at any degree of latitude, state thus: As radius at the equator is to 69.1 miles, so is the co-sine of the degree of latitude to the measure of a degree of longitude at the given degree of latitude.

Nat. sine of 90°.		Miles.		Nat. co-sine of 40°.
As 1.00000	:	69.1	: :	.76604

$$
\begin{array}{r}
69.1 \\
.76604 \\
\hline
2764 \\
41460 \\
4146 \\
4837 \\
\hline
52.933364
\end{array}
$$

Answer, 52.93 miles.

RAIL-ROAD SURVEYING.*

Preliminary Survey.

Sec. 179. A full party for the field operations of a preliminary survey is composed as follows:

One Chief Assistant Engineer,
" Compass-man or Surveyor,
" Assistant do.
" Leveller and Assistant Leveller,
" Rod-man,
Two Chainmen,
" Ax-men,
One Flag-man.

Sec. 180. With these the chief assistant goes into the field. He is supposed, of course, to have been previously made acquainted, by the chief engineer, with the general direction of the proposed rail-road, and some of the principal intermediate points through which the line is expected to pass.

Sec. 181. The principal objects of a preliminary survey are, to ascertain the distance between any given points, the difference of elevation between those points, and also the intermediate ground traversed, together with a general sketch and description of the different lines pursued in reaching the desired place.

Sec. 182. The distance and difference of elevation of any two points are necessary to enable the engineer to determine whether the rise or fall per mile, or the *grade*, as it is technically called, can be such as will render the motive power proposed effective. The bearing of the lines, together with the topographical sketches and field notes, are indispensable in determining the quantity or degree of the curvature.

Sec. 183. After being made acquainted with the general direction of the line, the chief assistant traverses the ground, examines it carefully for some distance in advance, and having determined upon the ground upon which he will run, directs a flag to some point as far from the place of beginning as it can be distinctly seen, and the

* This article was obligingly furnished by Engineer Sargent.

assistant at the compass takes the courses as in ordinary land surveying, laying off the line in stations of two hundred feet each, by means of stakes driven firmly in the ground and numbered, making No. 1, 200 feet from the point of commencement, and so on. During this process the chief assistant carefully sketches the topography upon each side of his line, directs bearings to be taken to the most prominent objects in the vicinity, notes the character of the soil, spring runs, or streams of water that cross the line, and determines the size of a sluice or drain necessary to pass the water. This done, the instrument is moved forward to the point where the flag stood, or some station in line with it, and the course tested by a back sight along the line of stakes to the point of commencement; when the same process is repeated, unless from the formation of the ground it is necessary to change the course. If this is the case, an angle is made of such magnitude as may be directed by the principal officer in the field; having due regard to the effect that will be given by tracing a circle between the two lines of which they will be tangents.

Sec. 184. The level follows the compass, using some point near the commencement as a base, and taking the relative heights of each station. This is usually performed by setting the level at station No. 2, and directing the rod-man to hold his target on the point established as a base, and to move the vane as directed until the leveller exclaims, *fast*. The rod-man makes fast and replies *fast*; when the leveller again looks, and if the horizontal hair of the instrument corresponds precisely with the middle of the vane, calls *right*. The rod-man then carefully reads from the rod the feet and decimals above the surface, and calls the result in a quick, sharp but distinct voice. This the leveller, assistant leveller, and rod-man, enter in their books, under the column of B-sights, and the rod-man moves forward to the next station and holds up on the surface, observing that he gets the natural range of the ground; when the observation is repeated as before, the result called, and entries made under the head of Fore-sights. Each then takes the difference and places it under the head *Above*, if the back sight is greatest, and under *Below*, if the fore-sight is greatest. The leveller calls the result, and the assistant and target-man assent or dissent, as their results agree or disagree with his. This done, the last Fore-sight is brought down

and placed under the head of *Back sight,* and an observation taken to the next station, the result of which is placed under the head of *Fore sight,* and the difference again taken ; and if the back sight is greatest, added to the last result, if less deducted. Again the last fore sight is brought down, and the same process repeated, giving the result for station No. 3 ; and the rod-man goes to No. 4, and directs a small pin to be driven, about six inches from the stake, firmly and close to the ground, upon which he places his rod for the observation, which when taken finishes the business of the "*set,*" and the leveller moves forward, takes his station at No. 6, and repeats the process before described ; unless, as is frequently the case, the undulations of the earth prevents his getting his sights from the regular stations. When this is the case, he avails himself of the most favorable position, having command of the most stations, and giving equal distances between the *peg* driven and the one upon which he again proposes to shift his level. The annexed table shows the manner of entering the field notes.

No	Distance.	Back Sight.	Fore Sight.	Difference.	Above.	Below.	Remarks.
1	200	8.204	3.30	+4.904	4.90		Start on the surface of the rail at the west end of Troy Bridge.
2	"	3.30	4.70	—1.40	3.50		
3	"	4.70	2.60	+2.10	5.60		
4	"	2.60	1.406	+1.194	6.794		—Peg.
5	"	9.465	6.54	+2.925	9.719		
6	"	6.54	4.75	+1.79	11.509		
7	"	4.75	5.302	—0.552	10.957		Bench No.1 on hickory tree west of line.
	"						

Note. It will be observed that the instrument has been changed; but the same process is necessary as in the preceding case, and the only difference is the relative position of the peg to the base first started with. The level from elevated positions is usually turned in various directions, to render it certain, the eye of the engineer has not been deceived in selecting the general route.

Sec. 185. After the field operations cease for the day, the leveller will examine his notes, comparing them with his assistant and rod-man, and foot his back sights and fore sights, to see if the difference correspond with that obtained in the field, also add or deduct,

as the case may require, to or from the starting point, and compare the result with that obtained at the point of leaving off.

Sec. 186. When a line for a rail-road has been traced as above described, the second step is to make a rough, or as it is termed, a working profile, and the engineer proceeds to adapt thereto the best grades it will admit of, and from the notes of the chief assistant, leveller, and compass-man, together with his personal observations, to suggest such changes, modifications, and improvements, as his judgment dictates, for the guidance of his assistant in executing the "*Definitive Survey*," which is the next step preliminary to breaking ground.

Sec. 187. The level in this follows the compass, as in the preliminary survey, noting every material deviation in the surface over which the line is traced; also, the level at the surface of water in all streams that are crossed, together with the soundings, and establishing frequent benches or permanent levels, on stumps, trees, or rocks, adjacent to, and convenient for the future adjustment of the line. The stations are now reduced to 100 feet, and when it is necessary to take intermediate levels, which is frequently the case, they are entered in the field notes thus:

No.	Distance.	Back Sight.	Fore Sight.	Difference.	Above.	Below.	Remarks.
40	100						
—	30						
—	40						
41	30						

Sec. 188. The duties of the level thus completed in the field, a second, more accurate and finished, profile is made, the grades adapted to it with the utmost care—the streams represented, also the division lines of farms, by a small flag or spear, and the name of the owner of each separate lot or farm, neatly printed between the boundaries, together with other general remarks, such as "opposite Bethlehem church," "road to the Shakers," "steam saw mill," &c. &c. Also the ratio of the grade per 100 feet and mile. The profile being thus far complete, and the grades satisfactorily arranged, the cuttings and fillings are next to be made out, and placed

along a line, drawn parallel to the base line of the profile—the cuttings being placed above, and the fillings below.

Sec. 189. The cuttings and fillings are deduced from the levels and grades in the following manner:

No.	Distance.	Surface.	Grade.	Cutting.	Filling.
470	100	246.57	250.87		4.30
71	"	246.71	251.71		5.00
72	"	260.00	252.55	7.45	

The grade being started at the base with the surface, is readily calculated from the rate which has been previously established. When the undulations of the ground are very abrupt, the intermediates are sometimes deduced in this survey, but not generally until the next

Staking Out.

Sec. 190. This consists in laying off the work and defining its boundaries, so as at the same time to procure the necessary notes for a correct measurement of the quantity of earth to be removed or supplied, and direct the contractor how to proceed with the execution of his work.

Sec. 191. The slopes necessary to be given in excavations and embankments, are determined from the nature of the soil. In embankments however, it is not usual (as we say,) to make them less than $1\frac{1}{2}$ to 1; that is, with a $1\frac{1}{2}$ base to 1 rise or perpendicular. These, however, vary greatly according to the views of different engineers, as well as from the circumstances above stated. We will suppose, then, that it is determined to lay off the road for a 15 feet width of bed on the top. Going, then, to station No. 10, we find it marked and also entered in our grade book—4.00 (4 feet below.) The instrument is placed and the rod sent to the nearest bench, which was marked +2.00—S. 10, (meaning two feet above grade at station No. 10.) The instrument was placed so as to command a view of as many stations as convenient, and the sight taken, which was 6.00. This was added, on a bit of loose paper, to the 2 feet,

making 8, and the letter T placed next the result, understood tripod, or that the cross hairs of the level were 8 feet above grade at station. The leveller then moved up his instrument to No. 10, and casting his eye to the right of the line, judged that the ground rose about 1 foot in 12, and gave the target-man the ring of the tape, and directing him to move off at right angles to the line, entered into the following calculations in his head: (Centre 4 feet.—1 foot rise leaves 3.—$\frac{3}{2}+3=4\frac{1}{2}$ and $4\frac{1}{2}+7\frac{1}{2}=12$.) At 12 feet then, the rod is held up and sight taken, which proves to be 12 feet. The tripod deducted showed the ground to be 4 feet below grade, and that he had not found the medium and was too near the centre, as running in his mind again the same calculation, he discovered that 4 feet fill would require $13\frac{1}{2}$ feet distance from the centre; so the rod was again held up at $13\frac{1}{2}$ feet, and the hair of the instrument cut the target in the centre, which had not been moved, hence gave the proper point of intersection with the surface of the proposed line of slope. The calculations previously gone through with in the head, were now made on paper by all the assistants comparing and agreeing; it was entered in the field book as shown in sec. 000. The opposite or left stake was set off in like manner, and the leveller and target-man moved on to station No. 11, and the grade ascending $\frac{40}{100}$ of a foot in a hundred, 0.40 was taken from the tripod, which gave 7.40 as the tripod for No. 11.

CURVES.*

Sec. 192. One of the most difficult parts of the field operations of a locating survey, and that perhaps in which more skill and judgment is necessary, than in any other particular case, is the changing the direction by curving, in a broken or hilly country; when attention must, at the same time, be paid to keeping upon a given level, or maintaining a uniform ascent or descent. Two methods are in common use for tracing curves. One by successive deflections with the compass or theodolite; the other by measuring offsets (secants) from tangent lines. The use of the compass is sufficiently explained in sec. 110, 111, and other parts of land surveying. If a theodolite is used, it should be of

* This article was prepared by engineer C. B. Evans.

the best make and graduated with the greatest possible care. If such is the case, a curve may be traced with a great degree of accuracy, in the following manner : Place the instrument on the station at which it is proposed to commence the curve, see that the zero of the nonius precisely corresponds with the zero of the graduated limb of the instrument, turn the instrument until the vertical hair exactly cuts the row of stakes and flag at the opposite end of the line, then tighten the clamp screw to secure the lower limb from moving, let down the needle and note the course of the line. You are supposed to be running on or near the line previously run in the preliminary survey, and therefore know about the number of degrees contained in the angle at which you are about to trace your curve. From this the quantity or degree of the curve is determined, and the curve, in technical parlance, is named from the number of degrees deflected from the course at each station, thus: if at each station the course is changed in the same direction two degrees, we call it a two degree curve ; if the course is changed three degrees, a three degree curve, and so on. Having determined what the degree of curve shall be, loosen the screw, which connects the moveable and stationary plates of the instrument, and turn the moveable part in the direction you wish to curve until the zero of the nonius corresponds with half the number of degrees on the graduated plate that you have determined upon as the quantity of the curve, thus: If you propose to trace a two degree curve, after setting the instrument upon the line as before, you will turn it round until the zero of the nonius cuts 1, or the first division on the card. Then let the chain be straightened from the station at the instrument, and a stake driven in line with the instrument as last set. This will be the first station in the curve. Then move the instrument until zero of nonius cuts 2° on the card, or one degree further than when the last stake was set, and at the end of another chain set another station, and for each station that you set while the instrument remains where you first placed it, you turn it one degree on the card. In this way you may set 8 or 10 stations in the curve without moving up with the instrument. But it must be borne in mind, that the farther you proceed from the instrument the more likely are you to vary from a true curve ; therefore, it is generally advisable not to set more than 8 or 10 stations without moving up the instrument. After setting

the stations as above, as far as can be seen with accuracy, move up the instrument and place it precisely over the centre of the last station set. Turn the instrument back again to zero, direct it to the first station back, and let down the needle to test the course. While the needle is settling, you may calculate what the course ought to be, as follows: Add together the number of degrees deflected in the curve, and add the amount if curving *from* the nearest magnetic pole, or subtract the amount if curving *towards* it, thus: The course of a line was N 20° W, and a curve 2° to the right, was commenced and run with the instrument nine stations, when it was necessary to move up.

Sec. 193. The instrument was carried to the last station and set as above directed. While the needle was settling the course was calculated thus: At the first station in the curve the deflection from the tangent was 1 degree, and the 8 following stations were 2 degrees each, making in all 17 from the course of the tangent. The bearing of the tangent was N 20° W, and the curve towards the north or the nearest magnetic pole; therefore, subtract 17°, the number deflected from 20 the course of the tangent, and it leaves 3° or N 3° W as the bearing of the last chord. On looking at the needle, the course as then indicated was found to correspond with that calculated, and therefore the work was known to be right.

Sec. 194. After directing the instrument to the last stake, tighten the clamp screw and turn the instrument 2° on the card for the next station; but for every station after that deflect but 1°, until the instrument is again moved up. When the course is sufficiently changed to stop curving, the instrument must be moved up and placed on the station which you propose to make the end of the curve. After directing the instrument to the last stake as before, deflect 1° on the card, and the theodolite will then be right for running a tangent to the curve. Set the stakes on the tangent as far as you can distinctly see; but before you move the instrument, test the course and see if the needle corresponds with what you make it by calculation.

Sec. 195. Running curves by offsets from tangents, depends upon the same principle as the one above described; but instead of deflecting with the instrument as above, departure equivalent to the degrees which would be deflected, is measured off from

the tangent or the chord of the last station, produced as the case may be, thus: If you wish to trace a 2° curve by measurement, you will produce the tangent one station farther than the one at which you wish to commence the curve; then, with a tape line graduated to decimals of a foot, measure off the departure for 1°, straighten the chain from the last stake to the end of the tape, and then set your stake. Then produce the line of the last two stakes a hundred feet farther, and from that point measure off the departure for 2°; bring the end of the tape to the end of the chain as before, drive your stake, and proceed on in this way to the end of the curve.

Compound Curves.

Sec. 196. The above methods describe a simple or regular curve, or a section of the circumference of a circle. It is sometimes necessary, after proceeding some distance with a curve, to change the degree of it, and curve faster, or not so fast, as the case may be; that is, a greater number of degrees are deflected at each station, or the curve made to conform to a shorter radius, thus: if after running some distance on a two degree curve, you find it necessary (to suit the formation of the ground, or from other causes,) to change your curve to 3°, you will proceed as follows: At the station where you wish to change make a deflection of two and a half degrees, and at the next three, and so on, as far as you continue the curve. In every case where you change from one degree of curve to another, add half the difference between the curve you are running and the one you wish to run, to the least, for the deflection at the point of change.

Reverse Curves.

Sec. 197. It sometimes becomes necessary to change the direction of the curve altogether, and without the intervention of a tangent, to change immediately from a curve in one direction, to one of a direction precisely opposite. When this happens, the operation is as follows: At the point where you wish to change, produce the chord as usual, but do not deflect; drive three stakes in line, and then commence deflecting in the opposite direction.

Pencilling and Calculating Curves,

Founded upon long Traverses through hilly Districts.

Sec. 198. After plotting an extensive traverse, taken for a road, and sketching some of the most important objects minuted in the field notes, proceed to pencil out the proposed road, so as to suit the eye or fancy. Then divide off the pencilled road into arcs; each arc extending as far as the curviture continues to be uniform. Then draw a chord line to each arc, and fix the point at each end by measuring from the nearest points in the surveyed traverse, if the ends do not fall upon any surveyed angle. As there will, probably, always happen an angle at some point in the arc, find the length of the chord line by the case in trigonometry, where two sides and a contained angle are given.

Sec. 199. Consider the angle as moved to the middle of the arc; for the angle will be the same in any part of the curve, according to a known principle in geometry. Double this angle and subtract that sum from 360; and the remainder will be the angle at the centre of the circle of the proposed arc, made by the radii limiting it. Connect these angles by a diagonal line (halving both of said angles,) and this will make four right angled triangles, each with a known horizontal leg; it being half of the long chord line. Find the length of the radius, as the hypothenuse, in the common manner of proceeding with similar triangles.

Sec. 200. Having found the radius of the arc, and the length in degrees, (which is the said angle at the centre,) find the measure of the arc in feet thus: Double the radius (making the diameter of the whole circle) and multiply the sum by the formula 3.1416—this gives the measure of the periphery of the whole circle. Then say, as 360° to the whole measure of the periphery; so the degrees of the arc (as expressed by the angle at the centre) to the measure of the arc in feet.

Sec. 201. The foot measure, and the degrees of the arc being known, divide the foot measure of the periphery by 100 feet, and the degrees by that quotient. This will give the number of stakes to be set, and the degrees of each isosceles triangle at its apex in the centre, for each portion of the periphery of the arc staked out.

SEC. 202. You are now ready to stake out the periphery into hundred feet portions (the usual practice.) Although these are chord lines, unless the curvature is too great for any rail-road or canal, such short chords will coincide so nearly with the curve, that they will come out about equal—or an allowance may be made by shortening the chain a few inches.

SEC. 203. The degrees for inflexion (or deflexion from the tangent,) at every stake, is to be conducted as follows: Find the direction of the radius, and set the compass on it. Then turn the compass around 90°, which brings it upon the *tangent line*. (The tangent line being always at right angles with radius.) Then deflex from the tangent line equal to half the angle at the centre of the circle, which is made by the radii limiting the 100 feet chord. But at every following stake, deflex equal to the whole angle at the centre; from the last line run.

SEC. 204. Fix the said tangent line as follows: The chord line will form an angle with a line in the traverse; from which the said chord line was calculated. This angle can be found in the common way for finding similar angles. Then sight the compass on said traverse line, and turn it through the number of degrees required for bringing the compass upon the chord line. Lastly, turn the compass through the number of degrees found by calculation, between the chord line and radius. This brings the compass upon radius, as required.

SEC. 205. Several other methods are in use among engineers. One is, to find the chord line of half the arc, and the versed sine, by trigonometry, instead of finding the radius as before explained. Then say: As the versed sine is to the said chord line; so is the said chord line to the diameter of the circle. Then proceed with the formula 3.1416, as before directed. Also halve the diameter to obtain the radius, to be applied as before.

SEC. 206. After having the general chord line, the chord line of half the arc, the radius, and angle at the centre, the periphery of the curve may be staked out by measures on the general chord line, and by offsets, as follows: The general chord line and the radii meeting each end of it, constitute a general isosceles triangle; consequently the base angles are known, according to division 4, of section 32. Each staked measure and the radii meeting each end

of it, constitute an isosceles triangle also, with larger known angles at the base. Subtract a base angle of the former from a base angle of the latter, and the remainder will be one of the acute angles in the right angled triangle, made up of a staked measured line as hypothenuse, and the offset, and run lines on the general chord line as legs; which two last lines can be found as in all cases of right angled triangles. After the first stake, the staked measured line will not form the hypothenuse for finding the offsets and run lines. But it must be found by doubling the first angle at the centre, and taking a radius for the middle term in the rule of three. In other respects proceed as before. Continue thus to calculate the run and offset to the middle of the arc; and apply the same measures for the other half.

Sec. 207. A curve may be staked out by running all the lines from one end, thus: The first staked measure begins at one end, of course. Find the next chord line from the end, as in the run and offset method, before described. All the chord lines may then be found in this proportion: As any one of the chord lines is to the sum of the two adjoining, so is any other chord line to the sum of the two adjoining. Substitute cypher for the outside chord line of the first measured line, for the purpose of uniformity, and proceed thus: Suppose P, at the point of beginning. Suppose Q R S T V W, at the terminations of all the chord lines, where the stakes are to be set in the periphery of the arc. Then say:

1st.	As	PQ	:	PO+PR	: :	PR	:	PQ+PS
		3.90	:	0+7.40	: :	7.40	:	14.04
								PS
				Subtract 3.90 from 14.04=10.14				
2d.		PR	:	PS+PQ	: :	PS	:	PR+PT
		7.40	:	10.14+3.90=14.04	: :	10.50	:	19.23
								PT
				Subtract 7.40 from 19.23=11.83				

	PS	:	PT+PR	::	PT	:	PS+PV
3d.	10.14	:	11.83+7.40=19.23	::	11.83	:	22.43
							PV
			Subtract 10.14 from 22.43=12.29				

	PT	:	PS+PV	::	PV	:	PT+PW
4th.	11.83	:	10.14+12.29=22.43	::	12.29	:	23.30
							PW
			Subtract 11.83 from 23.30=11.47				

The radius had previously been found to be 6.18.

Having calculated all the distances from the station at the end of the curve, the courses only are left to be found. Fix on the tangent line as directed in sec. 204. Run the first measure so as to form an angle with the tangent, equal to half an angle at the centre of the circle of the curve, made by two radii limiting said measured line. In this example the angle is , half angle. In running all the other lines, deflect half the said angle (as with the first measure,) from the last preceding course run. This will bring all the stakes to their true places in the periphery of the curve.

Sec. 208. The method of running on the chord line and making offsets, or ordinates, to places for setting the stakes in the curve, described in the last preceding section, may be calculated from these chord lines, PQ, PR, &c. Call each of these lines the hypothenuse of a right angled triangle, and suppose a vertical leg let fall upon the chord line of the whole arc, and you then have the acute angle at P; of course the run and offset (ordinate) are found.

Sec. 209. In running principal, or primary curves, the first method proposed (running on the periphery by inflections towards the centre—perhaps rather deflexions from the tangent,) is in general use. But in staking out the sub-curves, offsets, usually called *ordinates,* are chiefly used. The chord line of a sub-curve is now made, by most practising engineers, just one hundred feet in length. This is subdivided into portions of five feet each. *Ordinates* are thence set off to the places for the exact location of the curve. These may be calculated by one of the preceding rules, or by sec. 211 to 215.

SEC. 210. THE ORDINATES *for staking out the sub-curves,* being very numerous, the calculations are tedious. It is on this account that a 100 feet chord is assumed, as a common length for the chord of a sub-curve, and 5 feet as a common length for the distance between ordinates. This enables the engineer to calculate *a table of ordinates,* to fit all cases; and thereby to save much labor. In addition to this advantage, he avoids perpetual errors, which might be committed by less accurate assistants.

TABLE OF ORDINATES.

This table runs no farther than to the middle offset of the sub-curve, (usually called the versed sine,) because by inverting the order of the ordinates, the other half may be similarly staked out. The calculations for the lengths of the ordinates, are made to every degree and half degree of the first deflection from the tangent, from 1° to 14°. This deflexion is always equal to half the angle at the centre of the circle of the curve, made by the meeting of the two supposed radii, limiting a sub-curve.

TABLE OF ORDINATES.

Statements.	Primary Angle of Def. D.	M.	Ord. No. 1. Ft.	Ord. No. 2. Ft.	Ord. No. 3. Ft.	Ord. No. 4. Ft.	Ord. No. 5. Ft.	Ord. No. 6. Ft.	Ord. No. 7. Ft.	Ord. No. 8. Ft.	Ord. No. 9. Ft.	Prim'y Ver. Sin.
1	14	30	.60	1.12	1.61	2.02	2.37	2.65	2.87	3.03	3.13	3.16
2	14		.58	1.10	1.56	1.96	2.25	2.57	2.78	2.93	3.00	3.05
3	13	30	.56	1.06	1.50	1.89	2.21	2.48	2.68	2.83	2.89	2.94
4	13		.54	1.02	1.45	1.82	2.13	2.38	2.58	2.73	2.78	2.84
5	12	30	.52	.98	1.39	1.75	2.05	2.29	2.48	2.62	2.68	2.72
6	12		.50	.94	1.34	1.68	1.97	2.20	2.38	2.52	2.57	2.62
7	11	30	.48	.90	1.28	1.61	1.88	2.11	2.28	2.41	2.47	2.51
8	11		.46	.86	1.22	1.54	1.80	2.02	2.18	2.31	2.36	2.40
9	10	30	.44	.82	1.17	1.47	1.72	1.93	2.08	2.20	2.26	2.29
10	10		.42	.79	1.11	1.40	1.64	1.83	1.98	2.10	2.15	2.19
11	9	30	.40	.75	1.06	1.30	1.56	1.74	1.88	1.99	2.05	2.08
12	9		.37	.71	1.00	1.26	1.47	1.65	1.78	1.89	1.95	1.97
13	8	30	.35	.67	.94	1.20	1.39	1.56	1.69	1.78	1.84	1.86
14	8		.33	.63	.89	1.13	1.31	1.47	1.59	1.68	1.74	1.75
15	7	30	.31	.59	.83	1.05	1.23	1.38	1.49	1.57	1.63	1.64
16	7		.29	.55	.78	.98	1.15	1.28	1.39	1.47	1.52	1.53
17	6	30	.27	.51	.72	.91	1.07	1.19	1.29	1.37	1.41	1.42
18	6		.25	.47	.67	.84	.99	1.10	1.19	1.26	1.30	1.31
19	5	30	.23	.43	.61	.77	.91	1.01	1.09	1.16	1.19	1.20
20	5		.21	.40	.56	.70	.82	.92	.99	1.05	1.08	1.09
21	4	30	.19	.36	.50	.63	.74	.83	.89	.95	.97	.98
22	4		.17	.32	.44	.56	.66	.73	.79	.84	.86	.87
23	3	30	.15	.28	.39	.49	.57	.64	.69	.74	.76	.77
24	3		.13	.24	.33	.42	.49	.55	.59	.63	.65	.66
25	2	30	.11	.20	.28	.35	.41	.46	.50	.52	.54	.55
26	2		.08	.14	.22	.28	.33	.37	.40	.41	.43	.44
27	1	30	.06	.10	.16	.21	.25	.28	.30	.31	.32	.33
28	1		.04	.07	.11	.14	.16	.18	.20	.21	.21	.22

Sec. 211. To understand the table of ordinates, you should learn the method of calculating it. Proceed as follows: To have a clear view of the subject, plot an isosceles triangle. Make by any scale, tne oase 100 feet according to the practice of civil engineers in this country—this being a chord line of an assumed, or given, curve. Make the two equal sides at random; but at least three or four hundred feet. As the degrees of deflection of this chord, from tangent, must be given, [see sec. 206] double this and you have the angle of the apex of the isosceles triangle; which is the angle formed by the two limiting radii of the curve, at the centre of the circle, of which it is an arc. Then by trigonometry say: As the angle at the apex is to the hundred feet base line; so is half the remainder, after subtracting the angle at the apex from 180°, to the length of the radius.

Sec. 212. Double the radius gives the diameter of the circle of which the curve is an arc. Then add 50 feet (half the given chord line of the curve,) to half the diameter. Subtract that sum from the whole diameter; then multiply that sum by the remainder, and extract the square root of the product. This will give a standing ordinate, to be subtracted from all future ordinates to be obtained as hereafter directed; which remainders will be respective ordinates of the above table.

Sec. 213. The mathematical principle on which these calculations depend, is this: Wherever the diameter of a circle may be cut, the two parts being multiplied together, and the square root of the product being extracted, produces the ordinate erected on that point where the diameter is cut.

Sec. 214. As all the ordinates of the curve will be longer than the ordinate obtained, as in the last section, by the distance from the given chord of the arc to the centre of the circle; when it is subtracted from the ordinates extending from the centre of the circle to the curve, the remainder will be the length of the offset ordinates in the table.

Sec. 215. "The angle at the centre of the circle, of which the rail-road curve is an arc, is double the angle of deflection of its chord line from the tangent." This principal has been so often referred to, and is so important to the engineer, that (contrary to my general plan,) I will here demonstrate it.

Make the said chord line the base of an isosceles triangle; and let the radii, limiting it, be the two equal sides of the isosceles. And let the angle at the meeting of the radii in the centre (say 16°) be subtracted from 180°—leaving 164°. Halve this, making 82° for each of the base angles. Now it is manifest, that if 82° be subtracted from 90° (the angle formed by the tangent and the radius,) it will leave 8°, the angle between the tangent and the base of the isosceles triangle; which is the assumed chord of the curve (or arc) proposed.

Sec. 216. Cases may occur, where a curve may be required of the form of the long side of an oval. In such a case proceed in all respects as with a circle, until the radius and general chord line are found. Then shorten the radius by calculation, at the middle of the curve, as far as may be required. Consider the remainder of the radius as half the conjugate diameter of an ellipse. Also, the whole radius, doubled, as the transverse diameter. Offsets from the transverse diameter (ordinates) may be calculated thus: As the square of the transverse diameter, is to the square of the conjugate; so is the rectangle of the two abscisses of the transverse diameter (supposed to be cut where the offset stands,) to the ordinate or set-off. (See farther explanation of the ellipse hereafter.)

Sec. 217. Rail-road curves must not be too short, on account of the friction of flanges, and danger of running off; and experience must limit the highest admissable degree of curviture. They are generally compared by length of radius. But in absolute strictness they ought to be compared by a method, in part analagous to the principle on which the movements of planets in their orbits are compared—that is, similar areas with proportional lengths of arc. The arcs of the areas may be thus compared: Having first found the radius, length of the arc, and area, of the tried railway, double that area, and divide the sum by the radius of the proposed rail-way; which will give the length of the arc. The proportional lengths of the arcs will give the most simple and direct method of comparing them, with a view to their fitness,—the longest arc being proportionably the most curved and most objectionable.

Sec. 218. The Convexity of the Earth is such, that the *leveling instrument*, when pointing to a great distance, will cause a line to rise above the true level—that is, the line will be 7.9 inches far-

ther from the centre of the earth at the distance of a mile, than is necessary to constitute a level. In truth, by a level we mean a curve, which if continued, will form a circle around the earth, every where equi-distant from its centre. At the distance of two miles, the levelled line will differ from the water level of the earth 2 feet 7.9 inches—at four miles distance, 10 feet 7.3 inches—at eight miles distance, 42 feet 6.6 inches. These calculations were made on the supposition, that a tangent line nearly coincides with the 150 thousandth part of a circle (as 42 feet 6.6 inches amount to about that proportion of the earth's periphery.) Therefore this rule will be sufficient for all cases in practice. *Square the semi-diameter of the earth*, and the *superficial measure of the distance run*, separately—add these squares, and extract the root of the sum. This will give the length of a line from the centre of the earth to the levelled line (tangent.) Subtract the semi-diameter of the earth from that obtained line; and the remainder will be the perpendicular height of the end of said line.

Sec. 219. If the length of a degree of latitude (69.1 miles,) be calculated by the square root, according to the preceding rule, it will give 3211 feet 3.5 inches—whereas the true calculation by sines, &c., gives 3192 feet 9.7 inches. The error then in a degree of latitude will be about 18½ feet.

Sec. 220. If perfect accuracy is required in great measured lengths on the earth's surface, as 10 degrees of latitude, find the true length of the tangent line, and its elevation above water level, thus: Turn the measured length into degrees, by saying; as 25000 miles gives 360°, what will 691 miles give? Answer 10°. Now take this 10° for the angle at the centre of the earth, between the two radii, limiting the measured degree. This gives an isosceles triangle, with 10° at the apex and 85° at each of the other angles. Subtract the 85° from 90°, which gives the angle outside of the isosceles, between its imaginary chord-line base and the tangent. Then subtract the other 85° from 180°, which gives the angle outside of the isosceles triangle, between said imaginary chord line, and the secant extending the radius line up to the tangent. As the chord line at the base of the isosceles triangle is found by the given proportions of it, it follows, that all the angles and one side of the outside triangle being found, the true length of the tangent and radius

extended may be found. From the extended radius subtract the true radius, which gives the elevated end of the tangent with accuracy.

MEASURING EXCAVATIONS AND EMBANKMENTS.

Sec. 221. Under the head of mensuration, the method of calculating a parallelopiped, a pyramid, the frustrum of a pyramid, and a triangular prism (including the wedge,) were shewn. Engineer D. C. Lapham, has shewn us [Sil. Jour. v. 27, p. 128,] how to apply Professor Day's eighth problem in mensuration, so as to give a solution in cases, where part or all of these solids are found combined in a prismoid embankment, or in a prismoid mass of earth to be excavated. In truth it is a rule of most extensive generality, applying in all cases where there are straight sides; or where sides can be equibly averaged so as to approximate plane faces. Wagon-loads of coals with boxes sprung between the stakes, heaps of rough stone, ledges of rocks required to be cut down, basaltic hills, trunks of rivers, &c., may be calculated by it, with more accuracy than by any other hitherto discovered rule.

Sec. 222. Rule. *Divide the mass to be measured into so many sections, or prismoids, that each side shall be nearly straight from one end of each section, to the other. Find the area of the middle of each, and of both ends. Take the middle area four times, and each of the end areas once. Having added these six areas, multiply the sum by one sixth of the length of the section—this gives the solid contents.*

Sec. 223. Without giving a full illustration, it will be a leading thought towards an illustration, to refer to the wedge and frustrum of the pyramid. If the edge of the wedge is wider or narrower than the head, the width of the edge and of the two corners of the head must be added, to obtain an average of this modification of the triangular prism. Double the area of the triangular wedge is obtained by multiplying the length by the thickness of the head. Thus, having double the area, and three times the length, (calling it a triangular prism,) by multiplying these dimensions together we obtain six times the solid contents. Therefore it must be divided by six. The frustrum of a pyramid is a parallelopiped, four wedges

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and four pyramids. If the mass to be calculated is a parallelopiped, to take its area six times and then divide it by six, will produce the same result, as if but one area was to be used. But if the mass requires a wedge to be sliced off, to reduce it to a parallelopiped, the six areas will include the wedge, and not alter the calculation of the parallelopiped. As all masses with straight sides, &c., (and which can be reduced to such by judicious averaging,) may thus be calculated; this seems to be an exceedingly useful rule to the engineer. For it applies equally well to calculating the supply of water per second, &c., by a running stream—considering the *surface* of the water equivalent to the level *bottom* of a canal; and the *bottom* of the stream as equivalent to the uneven surface of a section of a canal.

Sec. 224. In taking the transverse areas of the masses to be measured, the levelling instrument is essential. A plain is assumed as the bottom of a canal or as the basis of a rail-road, &c., to be excavated; and its assumed depth is to be estimated from a fixed *chair*, (as it is technically called.) This consists of a stake driven strongly into the ground, or some other permanently secured object; intended as an index of reference, of a known height above the level of the bottom plain of the canal, &c., to be excavated. Stakes are set in the centre of the canal or rail-road ground.

Also in transverse sections, where areas are required to be calculated. These stakes have marks upon their levelled points, shewing their respective elevations and depressions, in relation to the object of their being set up. One hour's shewing, with the instruments in hand, is of more value than many days of reading. The manual use of instruments I shall not attempt to describe. A few general directions will be given in the proper place.

Sec. 225. In calculations for obtaining cross-areas, we rely upon these two *propositions*. 1st. Areas of trapezoids are found by adding opposite parallel sides, halving the sum, and multiplying the half sum by the distance between them. And, 2d, that a triangle made upon any line, will not change its area by moving its apex to any point on an opposite parallel line. [See sec. 33, 1 and 2.] If an excavation for a canal, diverging upwards, is to be made along a side-hill, so that the bottom will be but a foot or two below the surface of the earth at the lower side, and eight or ten feet below the

surface at the upper side, the transverse area may be found thus: 1st. Cut off a trapezoid at the bottom, up to the level of the surface of the earth at the lower side; and cast its area by the first proposition above. 2d. Find the level of the surface of the earth at the upper side; and multiply the upper side of the trapezoid by half the distance between it and the line of the level, which will give the area of all above the trapezoid.

Sec. 226. If the earth is undulating or ridgy, these rules may be applied so as to meet every case, with a little exercise of the inventive powers. For example: After taking off the trapezoid, as in the last section, if there is a ridge at the surface, take the level of its top and consider it the apex of a triangle whose base is the upper line of the trapezoid. Then if earth is left on one side of the apex, or even on both sides, the upper level may be considered as the base of a triangle, with its apex on the upper line of the trapezoid. And in some cases a trapezoid may extend to the level of the highest ridge, or knoll, and be cast as such. Then the vacant places be cast out or subtracted by making triangles or other regular figures, based upon the highest level.

Sec. 227.* The measurement of excavation, embankment and masonry on rail-roads and canals in this State, is usually reduced to cubic yards; while the latter item in some of the Middle States, is represented by *perches* of 25 *cubic feet each.* The method of determining on a side hill, the point at which the slope of excavation or embankment would meet the surface of the ground, was explained under the head of *Staking out.* At the same time, all notes necessary for the calculation of cubic yards in excavation and embankment, are taken and entered in the field book, as follows, viz.:

		Left.			Right.	
Stat.	Dist.	Dist.	Cut.	Centre.	Cut.	Dist.
40	100	14.60	+4.6	+3.50	+2.00	12.00
—	25	18.40	+8.40	+6.25	+4.62	16.62
41	75	13.00	+3.00	+2.38	+1.75	11.75

Sec. 228. When the ground is level, the point at which the slope

* The four succeeding sections were furnished by Engineer Evans.

will come to the surface, is found by merely adding the cutting of the centre to one half the width of the road, and laying off the distance at right angles from the centre, if in excavation; but if in embankment, add 1½ the centre filling to one half the width of the road, and lay off as before. This is correct only when the ground is a plain. For if the surface slopes transversely of the line, it is plain that the filling (if filling it is,) will be greater on the one side and less on the other, as you depart from the centre line. And if the filling is greater, the width of the base, or the distance from the centre to the outside of the embankment, will be increased in proportion to the slope of the bank—which we have before said, should be 1½ to 1 in ordinary cases. The object then of staking out, under the circumstances above described, is to ascertain the point at which the distance from the centre equals once and a half the filling at that point added to one half the proposed width of the road-bed.

Sec. 229. It frequently happens on side hills, that there is filling at the centre stake, but within a few feet the ground rises so much as to require cutting. When this is the case, it is necessary, not only for calculation but also for the convenience of the contractor in commencing work, to ascertain the distance from the centre at which the cut commences. Suppose the instrument set as before described under *Staking out*, and the elevation by grade at each station also known. Subtract the elevation by grade of the station in question from the height of the instrument, and set the target to correspond with the difference. Let the target-man then hold his rod upon the ground at a short distance from the centre, and move up the hill at right angles to the line, until the hair of the instrument cuts the vane, and the place where the rod then stands will be the point where the cut commences. The distance from this to the centre stake must be measured and noted.

Sec. 230. It has been a universal practice on public works, to require contractors to haul the earth a given distance from an excavation, before they receive pay for the same as embankment. This distance, however, is not by any means uniform, but varies greatly on works conducted by different engineers. On all our State canals, it is fixed at 100 feet; while on many of the rail-roads in this State, it is extended to 500 feet; but we consider a mean between the two to be preferable, and have therefore established it

at 300 feet. After calculating the whole amount of excavation and embankment; whatever of the latter item comes within this distance must be deducted.

CANALS.

Sec. 231. Under rail-roads I have included most of the calculations required for canals. These calculations I shall not repeat. Therefore, the student is to expect but little under this head which appertains to the mathematical arts, excepting what relates to items where water is an agent. This article will, therefore, be chiefly devoted to general descriptions; excepting that it will close with the necessary calculations on supply of water and filling and emptying locks. (See sec. 221—226.)

Sec. 232. *Navigation.* The general term for all transportation or conveyance by water, is navigation; which is divided into *natural* and *artificial.* But as natural navigation scarcely comes within the province of the engineer, I shall take no farther notice of this distinction.

Sec. 233. *Moving bodies on water.* The particles of water move over each other without much friction or with very little adhesion. Therefore heavy bodies move on the surface of water with little resistance. One man has moved 100 tons 7 miles in one day.

But as all heavy bodies sink into the water which sustains them, until they displace a measure of water of a weight equal to their own weight; it is manifest that a volume of water forward of the moving body must be displaced by it. It follows that the *form* of the body, and the place to which the water is to be removed, are important subjects for the consideration of the engineer. Also, that a boat carrying 50 or 100 tons, will not add one ounce to the pressure on an aqueduct bridge, while crossing it. The law which governs ship-builders, in giving form to vessels, is manifest in comparing the movements of a log or raft in water, with the sharp-built skiff or Indian canoe.

Sec. 234. Suppose the moving body, for example a crib-boat of lumber, to be 14 feet wide, and the canal the same width. Suppose the crib-boat sinks 3 feet into the water. The water before it is a wall 3 feet high; all of which must pass back by the stern, when

the crib is in motion, through the thin crevices on each side and beneath. The time required for this escape of the "wall of water" will be such as greatly to impede the progress of the crib. But were the canal 28 feet wide, the wall of water would escape laterally where there was but little resistance. Still the banks would present some resistance; as the waters nearest to the boat would be met by the waters stopped by the resistance of the banks. Hence it follows, that great breadth of water is favorable to the movement of heavy bodies on its surface. This principle is tested by our canal boats, when they pass by an artificial basin.

Sec. 235. *General law for constructing boats or other moving bodies.* The inclined plane and wedge are known to be mechanical powers, which give an advantage, directly as the length exceeds the breadth. Call the wall of water the resisting force, and horse-power, wind, &c., the power. Then the power will be to the resistance, as the greatest breadth of the boat to the distance from the place of its greatest breadth to the extreme fore-end, where it cuts the water. Consequently the longer the boat is, forward of its greatest breadth, the less power will be required to move it. But there are numerous other circumstances to be taken into view in practice, which every ship-builder understands. Such as, that the inclined plane principle applies to the ascent of the fore-part of the boat. For example, the scow, which retains its full breadth from end to end, has the advantages of the inclined plane in its ascending sled-like fore-end. Mathematicians have said that water would present a solid resistance when compressed with a velocity equalling 18 miles per hour. But this applies when a broad plain is presented; as if a plank-work should be fixed to the stem-piece of a steam-boat of equal area to a transverse section at the broadest part.

Sec. 236. *Choosing the ground for a canal.* In choosing the ground for a canal, where there is an opportunity to make a choice, the engineer should recommend to the directors the following considerations: 1. Course of the prevailing winds. 2. Kind of soil through which excavations are to be made. 3. Its termination in regard to commercial advantages. The illustrious Clinton told me he would not recommend a canal in the present age "which should terminate with its last excavation." Canals should be constructed for connecting navigable waters, or for connecting a navigable wa-

ter with a great coal-bed, mine, or quarry. Prevailing winds have not been duly estimated. Side winds, though favorable to all sailing craft, are very unfavorable to canal navigation. They always drive boats ashore, and give no assistance to its progress. Winds in the direction of the canal present an opposing force and an accelerating force, which counterbalance each other on the whole. Northerly and southerly winds being most prevalent in America, east and west canals are not so favorably situated in this respect. It is well understood by all boatmen, that winds are a less impediment (they are *always* an impediment) on the Champlain than on the Erie canal.

Sec. 237. Excavations made in plastic clay, marly clay, marine sand and crag, are found to be permanent. In truth, all stratified detritus, called tertiary formation, make good beds and banks for canals. The marine sand (bagshot sand) would seem to be unfit for canal embankments; but the trials at Irondequoit and Holley, on Erie canal, prove its fitness.

Diluvial and post-diluvial detritus are too variable in character for any general rules, excepting that ultimate diluvian and analluvian are good materials for canal beds.

Sec. 238. *Agriculture and health.* Canals should be constructed with a view to health and agricultural operations. For if they permit water to ooze through their embankments, health and agricultural operations are injured. Therefore diluvian containing vegetable matter should never be used.

Sec. 239. *Canal banks,* when they are not paved, should be bound by vegetables with creeping roots; particularly when the canal runs through diluvian, as from Oriskany to near Genesee river. Agropyron repens (quack grass) is probably the best of all American plants for this purpose. It will prevent the production of unhealthy gases, and prevent the banks from sliding down in the spring of the year. Banks are secured with well set paving stones where such stones are conveniently obtained. But the quack grass is best in wet places.

Sec. 240. *Stop-dams* were formerly made in canals each side of every place liable to failure; so that when it gave way the navigation would not be interrupted each side of the breach, nor the breach be enlarged by a long continuance of the flowing of water. These dams consisted of planks hinged to a bedded timber in the bottom of

the canal, so placed that a strong current would raise them up. Thus a breach in the canal by creating a current would stop itself. They are in some measure discontinued at the present time.

Sec. 241. *Aqueduct bridges* are bridges supporting canals which are carried over vallies, rivers, &c., the canals being constructed of wood or stone. They are but half the width of the canals, as the boats are never to meet on them.

Sec. 242. *Culverts* differ from aqueduct bridges in preserving the equal breadth of the canal, and in being constructed of earth, like the rest of the canal.

Sec. 243. *Waste-weirs* are openings on the sides of the canal, placed at a guaged height, so that the water will waste or flow out of the canal when it would otherwise be so high as to injure some part of the works. (See waste-weir under Water-Power.)

Sec. 244. *Tow-path* for men and teams to travel on when towing a vessel or raft which is floated in the canal. It should be three feet higher than the surface of the water of the canal; and a towing-rope should be 130 feet long. As teams must travel nights and days, and in rainy weather as well as in dry weather, the tow-path should be made of silicious or calcareous earth, that it may remain hard and even at all times. Its bed need not exceed three feet in width; therefore the cost of a solid bed will be repaid in one season of boating.

Sec. 245. *Cross bridges* are made for changing sides with the team, at places where the situation of the canal has induced the engineer to change sides with the tow-path. These bridges are narrow; and the tow-path is so arranged, that the teams cross the bridges without having the tow ropes cast off.

Sec. 246. *Heel-path.* The side of the canal opposite to the tow-path, is called (by way of a pun upon the word tow—toe) heel-path. The engineer should make the heel-path as good as possible without incurring much expense; for it is often almost as important to boatmen as the tow-path. Whenever a boat comes to for a night, or for a few hours by day, it must be on the heel-path side, and the boatmen will have frequent occasion to use it.

Sec. 247. *Location of locks* is a subject of great importance. In ascending ledges, and in some other situations, the location of locks is fixed as a matter of necessity. But in most cases, the location

of a lock is a very important subject for the consideration of the experienced engineer. When the ground is so nearly level that the locks may be at any places within two or three miles, they should be separated as near the usual distances for changing teams as possible. If locks could always be placed at the distance of six or seven miles from each other, this arrangement would be the most advisable. Long levels should never be sought. Every engineer concerned in laying out the Erie Canal, regrets having laid out the seventy mile and the sixty-two mile levels. By frequently agitating the water of canals by passing it through paddle gates under great pressure, atmospheric air is united with it, and gives it healthful briskness. The 83 locks of the Erie canal average four miles and a third from each other. The distance between many of them might have been proportioned better. For example, the nine locks at Cohoes Falls* should have been equally distributed to West Troy. This subject is at this day, (1838,) well understood; and the change is now going on, by averaging the locks along the high ground at the right of the present locks. It was found that the basins between the locks were so much limited in extent, that boats were often grounded by drawing off the water for filling the locks.

Sec. 248. *Form of locks.* The common form of locks is better than the elliptical form. And the size should never greatly exceed one boat in length and breadth, of the largest kinds which are to be used in the canal. For there is nothing gained by passing two boats at once, as the water required for filling is the same. Whereas there will be a great loss of water and time at large locks when but one boat is ready to pass. A still greater objection to large locks is, that larger and longer timbers are required for gates, which are heavy to move and more liable to be out of repair.

Sec. 249. *River locks.* It is generally advisable to construct locks on the side of a river, out of the reach of freshets, when they are required for conveying a vessel around a fall or rapid. Thus the sloop lock in Troy (New-York) should have been located. This would have kept sloops out of the current from the fall of water

* This article was published in 1830.

over the dam ; and when above the dam they would have been out of the influence of the water-fall, or draft of water as it is called.

Sec. 250. *A lock consists of side walls, fall walls, backings, wings, coping stones, recesses with hollow quoins, mitre sills, a pair of gates at each end,* and each gate containing *paddle-gates* and a *balance beam.* Some locks have a *crooked culvert* in the wall for each paddle-gate ; and some have siphons in lieu of paddle gates. Two or four paddle-gates are most approved; as one may be opened while the water is low before the gates, and the other after the water has risen to the paddles. Besides a paddle-gate way will weaken the main gate if large enough to let the water through expeditiously ; and it cannot be opened without great power, if large.

Sec. 251. *Laying out a canal.* This may be conducted in all respects like laying out a rail-road, as directed under that head ; so far as respects all mathematical calculations and operations. In general there is not as much care required as to accuracy in turning curves, and minute levelling by the inch or foot. But the line must be straight to a considerable distance each side of every cross bridge, aqueduct, lock, or whatever lessens the breadth of the canal, by which boats would be liable to strike.

Sec. 252. *Reservoirs.* If there is great necessity for depending on water for the summit level which will be deficient in the dryest seasons, the line should be so run as to afford a convenient place for a reservoir of sufficient capacity. The reservoir should be a very little higher than the canal level ; so that all the water not essential for practical use, may be saved in the reservoir to let into the canal from time to time as occasion may require.

Sec. 253. *Water for supplying feeders.* In laying out a canal the most important consideration is a sufficient quantity of water for feeding it. To ascertain whether or not the supply will be sufficient, observe ths following directions: How many boats will probably pass every day ? For a column of water whose base is equal to the square of the area of the lock, and whose height is equal to the difference between the highest and lowest surfaces in the lock, will be required for every boat that passes to the lower level. But we may suppose that only one half of the ascending boats will require the same quantity. Because if a boat ascends first after one has descended, no water is to be taken into account.

Sec. 254. Allowance for filtrations and evaporations should be equal to about one-twentieth part of all the water used for feeding the canal.

In addition to all the allowances made for the first lock on each side of the summit level, allowance must be made for each succeeding lock, which is supplied from the summit level. Perhaps one twentieth for each lock. Two methods are in use for supplying the succeeding locks: 1st. To construct a sluice-way by waste-weir, to carry the excess of water around the upper locks to supply the waste to the lower. 2d. To make every succeeding lock to fall about six inches short of the preceding, in depth.

Sec. 255. *Supply of water by feeders.* The quantity of water which passes any point in a flowing stream per second, must be ascertained before a decision is made in regard to its sufficiency as a feeder for a canal. A calculation must be made of the supply of water which the stream will afford when it is at its lowest, in summer droughts. The best method of calculation is, to consider the surface level of an assumed trunk of the stream, as equivalent to the bottom level of a canal excavation. (See sec. 221.) Then take transverse depths of the stream, and consider them as the upper levels for calculating transverse areas of excavations, as taken by the level. (See sec. 222.) Then proceed by the four middle areas, and the two end areas, in all respects as directed in excavations and embankments. (Read sections 221, 222, and 225, attentively.)

Sec. 256. The cubical contents of the trunk of water being found as if it was a permanent solid, the time it occupies in passing over its lower limit must be ascertained. If the stream does not exceed about three feet in depth, branching limbs with plenty of leaves may be thrown into the upper end of the trunk, and repeated several times, for determining the velocity. The branches must occupy nearly the whole depth, as water flows faster near the surface. They must be thrown in so as to average the velocity from near the shores to the centre. In deep rivers an empty and filled bottle may be tied together, so that one may float and the other sink.

The velocity may be measured by a watch; but a second pendulum 39.1 inches in length, or a half-second pendulum 9¾ inches is preferable; or 9.77, more accurate.

Sec. 257. Having obtained the result of the calculations of the two last sections, proceed to compare the quantity of the supply, with the quantity required for filling the locks the maximum of times required; allowing the full measure for each filling, and also allowing for all the wastes described in sec. 254.

Sec. 258. *The time of filling and emptying locks* is not connected with the general supply, any farther than to determine whether it will be too long for a reasonable delay of boats. But the paddle-gates for the admission and discharge, is a necessary subject of calculation. On referring to sections 139 and 140, and by an application of common sense, this calculation will be evident. But the discharge may require more particular calculation; therefore a full description of a familiar example will be found in next section.

Sec. 259. Calculation of the time of emptying the east lock of the pair of locks at the junction of the Erie and Champlain canals.

	Ft.
Length of the lock within,	89
Average breadth within,	15.5
Depth,	14

```
  15.5        217
    14         89
  ----       ----
   620       1953
   155       1736
 -----      -----
 217.0      19313 cubic feet of water contained
                  in the lock.
```

Four paddle gates in the lower gate, each two feet square; making each an aperture of four square feet.

Reduce each aperture one third, on account of friction (adhesion) and contraction of vein.

```
3)4.00
  1.33
  ----
  2.67 Reduced aperture.
```

Head is 14 feet; but falling water diminishes the force of the head one half. Therefore the head is to be computed as if 7 feet high.

7.0000(2.65	2.65	Square root of the reduced head.
4	8	Velocity of one foot head.
46)300	21.20	Length of effused trunk per second.
276	2.67	Reduced aperture.
425)2400	14840	
2225	12720	
	4240	
	56.6040	Water effused each second from each
	4	paddle-gate.
	226.4160	Effusion per second from all the gates.

226.4)19313.0(84 Seconds, true answer.
18112

12010
10056

Sec. 260. The calculation made in the last section, was from measures and trials actually made in January 1838, by the engineer class of Rensselaer Institute. By starting all the four paddle-gates at once, and measuring time by the oscillations of a pendulum 39.1 inches in length, they emptied the lock several times, in precisely 84 seconds each time. By often repeating similar measurements and calculations, a very important branch of engineering will become familiar. Flumes of flouring mills, factories, &c., are calculated in this manner; excepting that the square root of the whole head is taken, as in all cases where the water is kept at a uniform head. (See sections 139 and 140.)

ROADS IN GENERAL.

Sec. 261. *Kinds of Roads.* The principal distinctions among the roads of this country are, 1. *Unbedded*, most common roads, which receive their forms from the feet of horses and wheels of carriages; or remain as they were when first laid out. 2. *Turn-*

piked, when made of earth in the form of beds, descending into lateral ditches. 3. *McAdamized,* when made in the same form of turnpiked roads; but the materials consist wholly of pounded stone, included between curbs.

Sec. 262. *Laying out roads* is too often under the influence of private interest; otherwise they might always be well laid in newly settled countries. But it is very difficult to change the location of a road in an old inhabited town. The ground chosen for common roads, to be supported by tax, should be free from sloughs, and should avoid clay-beds as much as possible. This should be done at the expense of distance; for the taxes are generally too low for raising large sums for bedding and repairing roads through such places.

Sec. 263. *Causeys and bridges.* So many treatises are before the public on these subjects, that little remains to be said. They should never be at the foot of a steep hill, when it can be avoided by turning the road, or by digging down the hill. And causeys should not be made with large logs or large stones. For the earthy covering will soon be worked down among such coarse materials; leaving them a naked nuisance. If large stones or large logs are laid in, and covered with small stones or saplins, the evil will thus be remedied.

Sec. 264. *Breadth of causeys and bridges.* On the score of economy, as well as of convenience, these should be wider than the custom is, in this country. Bridges will stand much longer for having wide abutments; and causeys (especially if made of wood) will be more firmly fixed, if wide.

Sec. 265. *Mile and guide-boards.* These should never be made of stone. Good sound plank are better. Every one has observed the destruction of mile-stones by mischievous villains. Plank cannot be destroyed, nor even injured, without considerable labor. But a mile-stone is broken and destroyed by one stroke of an ax, or one stroke with a large stone. Guide-boards are always made of wood; but a mere slip of a thin board, nailed to a post, is soon demolished. A single plank when but two roads meet; or a triangle or quadrangle of planks when more than two, should be used, half-charred at the ends set in the ground.

Sec. 266. *Planting, or leaving, trees.* Trees should be left in laying out roads through woods, and should be planted out in all other cases, in such situations that the road may be shaded from 9 A. M. to 4 P. M. from 1st May to 1st September. The best indigenous trees in America are the red maple and sugar maple. Trees are of more value than is generally supposed. Take the following calculation : 200 trees may be set at 25 cents each—that is, the 200 trees per mile will cost $50. The cost will be $2000 for 40 miles, a day's travel for a team. On all great thorough-fare roads, 40 loaded teams will pass each day, for 150 days of hot weather. This gives 6000 day-journeys ; which would be but 33 cents each, if the whole expense must be paid the first year. If the public fund only is considered, we may safely say, that less than half a cent is paid each trip for the benefit of dense shades for 40 miles, in the oppressive heat of summer. For all such trees will endure 60 to 80 years.

Sec. 267. *Watering places.* The value of watering troughs far exceeds that of shades. Walking horses, which draw loads, should drink once in three miles. Watering places must be supported by wells, excepting in places where there happens to be a stream. To neglect watering places ought to be made a crime by statute, for which road commissioners should be indictable.

Sec. 268. *Level roads.* For loaded carriages when the horses walk, a level road is best. But for trotting teams a road is best, when moderately undulating. Even a hilly road, in such cases, is better than a level one. For a load pressing alternately on breast and breech, is easier for the horse.

Sec. 269. *Resting places.* Loaded teams are greatly relieved by resting places on hilly roads. And even pleasure-carriage travelling is often benefitted by them. They are transverse mounds of earth, made smooth and sloping ; a little oblique to the direction of the road, that they may serve to direct the water of sudden showers into one of the ditches.

Sec. 270. *Zigzag roads.* Such roads are required on the face of steep mountains. Such a road takes seven tacks while ascending Catskill mountain to the lakes and Mountain House. The principal subject to be considered in laying out such roads is, to give broad spaces for the turns. I laid out that road, under the direction of the

commissioners. At first without any regard to the breadth of the turning ground. But I was compelled to alter the whole arrangement, after the work was commenced, in the year 1807.

Sec. 271. *Dug-ways.* When roads are cut into side-hills, like shelves, they are called dug-ways. One rule is never to be overlooked in such cases. It is, that the outer side of the road, from the hill, must always be about one foot higher than the inside. And it must not be forgotten, that all springs issuing from the hill, must be carried across, under the road, in very large sewers. This is necessary to prevent an accumulation of ice-ledges across the road, which make it totally impassable.

Sec. 272. *Agriculture and health.* The same rules apply to roads, which are applied to canals in section 238. It may be added that roads are so numerous, as to furnish the means of doing much good in this respect. All the individuals in society make up the whole of community. Therefore if the property of one is benefitted the whole body is benefitted. Numerous cases occur to an honest board of road-commissioners, for doing much good. All conductors of water may be directed so as to benefit the nearest farmer, without any injury to the public. So in passing farm houses, there are numerous ways for an accommodation. Stagnant waters should never be allowed to settle down from a road near a dwelling house.

Sec. 273. *Hills and mountains.* The ascents and descents of hills, are important subjects for road-commissioners. By attending to the laws of the inclined plane, and to the balancing principle applied by the horse in drawing a load, common sense will suggest rules of practice. When a horse is drawing to the extent of his strength, his hind feet form a pivot upon which the weight of his body is balanced against the resistance of the load. Should the hill be so steep, that the centre of the gravity of the body of the horse, is directly above his hind feet, he can draw nothing. Reduce the steepness of the hill, and the weight of the horse will apply in an increasing ratio. This ratio will have the advantage of the lever also. The ascent of 18 inches to the rod is the limit imposed by the legislature of the State of New-York, on several turnpike companies in mountainous districts—that is, an ascent of 5 degrees. If the ascent exceeds 6 degrees, it is a tiresome road; and commissioners

ought to avoid any farther increase in the ascent, by the zigzag form. (See sec. 270.)

Sec. 274. *Angle of friction in the movement of carriages on different roads.* The angle of friction is estimated by placing a carriage on an inclined plane in the road to be tested, where the descent is just sufficient to give the smallest degree of motion. The motion must not be sufficient to acquire any increased velocity by its progress. A section of this inclined plane is to be considered as radius and the height of the elevated end as the sine of the angle of ascent. One ton for a load on a M'Adam road, will move with the radius 50 to a sine of one by measure—on a very smooth pavement the radius of 68 to a sine of one. On a level road of the same quality, the same proportional of weight, suspended over a pully, as the sine to the radius, will start the load. Thus about 40 pounds suspended over a pully will start a ton load on a level, if the carriage is of the best construction in regard to friction.

Sec. 275. *Location of bridges.* Two considerations must always govern in the location of bridges, if any choice can be exercised. 1. It should be located below a natural ice-break if possible; as a fall of water, &c. 2. It should be placed where the abutments may be secure; for where abutments can spread, the bridge, if arched, is never secure.

Sec. 276. *String-pieces.* When a bridge is made by planking upon straight string-pieces, they must be strongest in the middle. To be largest is not always to be strongest. If the grains of the timber are straight, and the top of the string-piece straight on the top, it will be but little stronger for swelling underside so as to be much thicker in the middle. For its increase in strength depends on the lateral adhesion of the fibres, which is feeble in the straightest and most thrifty growing timber. But if the timbers swell out at their sides, this objection will not apply in so great a degree.

Sec. 277. When string-pieces are supported on bents or piers, some calculation is required in setting off the distances between the supports, or in selecting timbers of the most suitable length. By referring the mind to stations taken by men, when carrying long and heavy timber, common sense will have a sufficient guide in this matter. For example: 6 men are to carry a heavy beam, which is 60 feet long. To place these men in pairs, so that each pair may

support equal portions of the weight, their carrying-sticks must be so placed that each pair shall bear 20 feet. If the timber was cut into three 20 feet pieces, and each stick was put under the centre of each 20 feet, it is manifest, that the weight of the whole 60 feet would be equally distributed. To arrange the three pieces in one line, touching end to end, would not alter this proportion—neither would pinning, or otherwise uniting them. Therefore the first pair would be placed 10 feet from the fore end—the second 30 feet—the third 50 feet. If two of the men were placed at the fore end, and the other four were to lift at one stick, that must be placed 45 feet back. This would leave 15 feet back to balance the 15 feet between it and the centre. The pair at the fore end, would, upon the lever principle, lift but half the other 30 feet, if the back half was cut off and the other stick placed at the new cut end. But after all the back half is neutralised by the balance of the part back of the hind carrying-stick, the two sticks are applied to the fore half with different levers by 15 feet. That is, the centre of the weight of what is not balanced, is 15 feet from the forward carrying-stick, and 30 feet from the back carrying-stick. Therefore the forward stick will support two thirds (20 feet) and back stick one third (10 feet) which, added to the back 30 feet, gives the true proportions.

I preferred this method of illustrating the whole doctrine of beam pressure to that of giving a set of rules. Not only bridges, but flumes, mill-dams, locks, house-beams, &c., &c., require an attention to this subject.

Sec. 278. *Repairing roads.* It is astonishing that our highway masters and turnpike companies, still continue to fill up ruts with loose soil; when they see the first wheel that follows such repairs restore the ruts. It is still more absurd to fill ruts with stones, wood, &c. But one efficient method has hitherto been adopted. It is to fill the ruts with the same material of which the road consists, by pounding it down in a succession of thin layers, until it is considerably harder than the rest of the road. This makes a durable repair and the labor is not great. Two men with sledge-hammers, one on each side, will pound the fourth of a mile of bad ruts in one day; while one man with a cart and one with a spade will supply the filling.

WATERWORKS.

Sec. 279. The term *Waterworks* is applied to the conduction of water through pipes or raceways, (mostly through pipes,) where water is to be used as an element; not for its mechanical force. The principles of waterworks, as a science, are not generally studied, and, of course, are little understood.

Sec. 280. We have but one American treatise; and that has, probably, no equal on either continent, as a concise digest of all that is valuable on the subject. I mean *E. S. Storrow's Treatise on Waterworks.* To those mathematicians who wish to go deeply into the subject, and to trace this truly experimental science back to its origin, then to follow down its history to the present day, this little duodecimo of 242 pages, is most emphatically recommended. He does justice to the early investigations (in 1771) of Abbe Bossus—of Chezy (1775)—of Dubuat (1786)—of Coulomb (1800)—of M. de Prony (1804.) But the German, Eytelwein, may be said to have given the last finish to the formulae now in use, in 1814 and 1815, through the Memoirs of the Academy of Berlin.

Sec. 281. Waterworks being a subject not commonly studied by American engineers, I shall here give a few essential formulae, with a very general statement of the theory.

Sec. 282. Water moves in pipes or raceways, under the government of ACCELERATING FORCES and RETARDING FORCES. In strictness, there is but one *accelerating force;* which is the HEAD WATERS above the place of DISCHARGE. And there is but one *retarding force;* which is the ADHESION against the sides of the conducting PIPE, OR RACEWAY. But in calculating practical results, it appears to be necessary to take into view the *area* of the pipe, or raceway, the interior *surface* of adhesion, and the *length* of the raceway, or pipe.

Sec. 283. It has already been shewn [see sec. 139 and 140] that the force given by the head waters, is as the square root of the height. But the retarding power of adhesion (sometimes called friction) depends, for the estimate of its influence, solely on trial. As different calibres of pipes, and different measures of sides and bottoms of raceways, present different proportionals of surface for ad-

hesion, nothing but very extensive series of experiments could furnish rules, or formulae, for practical use.

Sec. 284. Though the fall of water (that is, the elevation of the head above the place of discharge,) is elementarily the only *accelerating* force, and adhesion to inner surface of the conducting pipe or channel, is the only *retarding* force; yet it is found that several modifications and combinations of these elementary principles, must be taken into all calculations—for the reasons read Storrow.

Sec. 285. In pipes of wood, iron, earthen, or whatever close conductors may be used, the following combinations and modifications are necessary:

1. Diameter of the pipe.
2. Difference of level between the head and discharge of the water.
3. Length of the pipe—consequently the continuance of adhesion.
4. Difference of level divided by the length of the pipe.
5. Velocity per second. Here each section is considered as the acquired velocity, or the successive retarding results; and may be estimated in infinitely small divisions.
6. The quantity discharged per second in cubic feet.

The area of a section of the pipe, as divided by the inner perimeter, requires some calculations. This quotient is called the *mean radius*. It may be perceived, that the larger the area the greater will be the velocity, but the greater the perimeter the less the velocity. In truth, after the perimeter is increased to a certain proportion, compared with the area, it totally overcomes the area; and the water stops flowing, by the principle called capillary attraction.

Sec. 286. *Formula for pipes when quantity discharged is sought.* The engineer is called on to answer this enquiry: How much water will be discharged per second, if the head (above the place of discharge) is 70 feet, the diameter of the pipe 10 inches, and the length of the pipe 900 feet?

Prepare for the rule by reducing the feet to inches.

Rule. 1st. Multiply the diameter by 57, and that product by the head (70 feet) and set this last product down for a dividend. 2d. Take the first product (diameter multiplied by 57) and add to

it the length of the pipe (900 feet) and set this sum down for a divisor. 3d. After the division, extract the square root of the quotient. 4th. Multiply that root by 23.33—the product will be the velocity in inches per second. 5th. Find the area of the pipe at the place of discharge, and multiply that by the velocity in inches per second. This product is the quantity of water discharged per second in cubic inches.

SEC. 287.* *Formula for pipes when the diameter is sought.* What diameter of pipe will be required to discharge 3 cubic feet of water per second, if the head is 10 feet and the length of the pipe is 4000 feet?

Prepare for the rule by reducing all the measures to feet and decimals of feet.

Rule. 1st. Square the cubic feet (3) multiply the square by the length of the pipe (4000 feet) and take this product for a dividend. 2d. Multiply the head by the square of 38.116, ($10 \times 38.116 \times 38.116 = 14528.29$) and take this product for a divisor. 3d. After the operation of division, extract the root of the quotient to the fifth power, by sec. 30; which will give the diameter of the pipe in feet and decimals of feet (1.199.)

SEC. 288. *Formula for open canals when the velocity and quantity are required.* How much water will be delivered per second, if the area is 4.8 feet (2.4×2) head 10 feet, perimeter inside 6.8, length 30 feet?

Prepare for the rule by reducing all the measures to feet and decimals of feet.

Rule. 1st. Multiply the area and head together for a dividend. 2d. Multiply the perimeter and length together for a divisor. 3d. After the operation of division, multiply the quotient by 9582, and to this add 0.0111. 4th. Extract the square root of the last sum. 5th. To the root add 0.109. This will give the velocity of feet per second. Multiply the said feet by the transverse area of the trunk of flowing water, which will give the quantity in cubic feet.

SEC. 289. As open canals present their flowing waters to the eye, their laws of motion are subject to more convenient inspection than those of pipes. At the Rensselaer Institute the students in civil en-

* This, and three other examples, were obligingly calculated for my pupils by my learned friend, Mr. Storrow.

gineering have generally obtained the following proportional results, or nearly so. A raceway, smoothly planed within, with exact sliding gates 36 feet apart, is used. It is 2 inches wide and $2\frac{1}{2}$ deep within. When the raceway is so inclined that the upper gate is 56 inches higher than the lower one, the water flows from gate to gate (36 feet) in 6 seconds. If the upper gate is drawn 2 inches, the velocity so far contracts the flowing trunk of water, that the lower gate precisely touches the surface when it is drawn 1.1 inch. Therefore the increased velocity, under an angle of 7° 23′ inclination, diminished the volume of water nine twentieths in flowing 36 feet.

SEC. 290. These experiments may not be perfectly accurate. But they approximate truth near enough for general illustration; and students of all schools should repeat them. The law of *falling bodies*, as illustrated in sec. 140, and the law of the *inclined plane*, as in sec. 152, should be referred to in explanation of this experiment. Students will not overlook the difference between pipes and open raceways, caused by the water on the upper side, in the latter case, not being subject to adhesion. Of course the perimeter includes the bottom and two sides only; whereas pipes present adhering surfaces on all sides, with additional resistance from adhesion on account of increased pressure against the whole inner surface.

SEC. 291. *As atmospheric pressure* often has more or less influence upon the flowing of water in pipes and open raceways, the student is referred to sections 141—144, where the most important principles are explained. The pressure averaging about a ton weight to a square foot near tide-water level, it often becomes a subject deserving particular attention. But on high mountains the pressure of the atmosphere is greatly diminished. It even becomes so rare at the height of about 45 miles, that it does not reflect the sun's rays sufficiently to become visible in the state of twilight.

SEC. 292. The highest point where the atmosphere is sufficiently dense to reflect light, may be found as follows:

1st. Take the *time*, by a good watch, between the disappearance of the sun in the western horizon, and the disappearance of twilight.

2d. Calculate this *time*, in the manner hereafter explained, so as to ascertain what the time would be (or is) where the sun goes down vertically (as at the equator on the 22d of March.)

3. Then say, as the minutes of 24 hours to 360 degrees, so are the minutes, *vertically taken*, between sun-set and twilight-set (about 70) to an angle at the centre of the earth, formed by a radius to said centre from the observer, and from the place on the earth where the sun disappears, where the observer sees the last departing ray of twilight in his horizon.

4th. Take half said angle at the centre of the earth, for one of the acute angles of a right angled triangle, formed of the observer's horizon, his vertical semi-diameter of the earth, and a line from the centre of the earth to the point of the last appearance of twilight.

5th. Then say, as the co-sine of the half angle at thé centre of the earth, is to the semi-diameter of the earth (about 4000 miles,) so is radius (the angle at the observer) to the distance from the centre of the earth to the point of the last appearance of twilight.

6th. Subtract from the last answer the semi-diameter of the earth (4000 miles) and the remainder will be the height of the atmosphere, where it is just dense enough to reflect and refract the sun's rays sufficiently for rendering it visible to the earth's inhabitants.

Sec. 293. It may be desirable to the correct student, to understand a practical method for determining the true time to be assumed for the calculations of the last section, between the disappearance of the sun's face, and the disappearance of its last departing rays of twilight. The plainest practical method, and the one best adapted to student's practice, is as follows :

1st. Set a compass to find the bearing of the point of the sun's setting, and note the time of its setting.

2d. In the same manner, note the time and bearing of the obtuse apex of the last departing rays of twilight. Thus you have the difference of time between the disappearance of the sun's face and of twilight.

3d. Call the difference of time between the setting of the sun and the setting of twilight, the hypothenuse of a right angled triangle. Call the horizontal angle between the point where the sun sets and where the day-light sets, the horizontal leg. For the vertical leg, (the answer required,) apply the rules for right angled triangles. Also remember to turn time into angles, as in all cases where 24 hours give 360 degrees, &c.

Sec. 294. The said vertical leg may be found without taking the bearing of the points at setting of the sun and twilight, by finding the angle which the sun makes with the horizon at setting. This may be done by calculation, made from the latitude of the place of observation and the sun's declination.

Sec. 295. As aqueous vapor diminishes the specific gravity of the atmosphere, it often becomes a subject of consideration—particularly in the use of the barometer, and in calculating for the ascent of water in pipes in passing over hills, &c. Vapor being visible in the form of *clouds* or *fogs*, it may be well for the student to give his attention to the *natural history* and *heights* of clouds, for part of each day during one week; as clouds are lighter than air.

Sec. 296. *Five forms of clouds often precede each other in regular series.* In fair weather during summer months, the *stratose* clouds, usually called fogs, often appear in the morning near the earth. After the sun shines upon them, they ascend in a state scarcely visible, and at length form the *cumulose* clouds. These are the bright shining clouds in brilliant heaps above, with apparently straight bases below, when viewed horizontally. They ascend still higher later in the day, and form the *cirrose* clouds. These have a fibrous flax-like appearance, and rise the highest of all clouds. At length they descend more or less, and become the *cirro-cumulose* clouds, by assuming a knotted or curdled appearance at first, and then becoming confluent. They either become stationary, producing rain or snow; or break up, and their fragments become *cirro-stratose* clouds. These are the patches which have a stratified appearance when viewed horizontally; but they never approach the earth, like fog. No rain falls from this series of clouds, excepting while in the *cirro-cumulose* form.

Sec. 297. *Three forms of clouds seem to be independent of all other forms, and of each other.* The *nimbose* cloud, generally called the thunder cloud, as soon as it commences forming, begins to move pretty uniformly and steadily. At first it exhibits a heaped top, like the cumulose cloud; but as it advances in size, it shoots forth a kind of spray-like form from its uppermost heads. It usually produces rain, and breaks up soon after. The *villose* is a kind of open fleecy cloud, called scud, which moves with great rapidity, often in a direction different from the clouds above. It is generally

formed suddenly, and breaks up suddenly. The only remaining variety is the *cumulo-stratose* cloud. It is very rarely formed, and always appears to rise up in the horizon like the smoke from a furnace. Its top generally seems to pass into a *cirro-stratose* cloud above, and there spreads out like the top of a mushroom ; it is therefore generally called the mushroom cloud.

All snow storms and settled rains proceed from the *cirro-cumulose,* and all hail storms and showers, from the *nimbose,* clouds.

Sec. 298. *The height of a nimbose cloud* may be taken as shewn in the following example : May 30th, 1837, during a severe thunder shower, I suspended a pendulum near the west door of the Institute, and directed an assistant to watch its vibrations, while I observed the origin of three successive chains of lightning. The assistant noted, by the pendulum, the seconds between the flashes and the sound. The time averaged $21\frac{1}{2}$ seconds—and the angle above my horizontal level was found, by the sextant, to be $11\frac{1}{2}°$. Allowing 1124 feet per second for the sound, the hypothenuse from that point in the cloud was 24166 feet. This gave the height of the cloud 4818 feet above my level. The earth's convexity (after finding the horizontal leg) gave 13.3 feet. (See sec. 218.) Therefore the height of the cloud was 4831.3. But my level was 73 feet above the tide-water of the Hudson—of course the cloud was 4904.3 feet above tide-water level.

WATER-POWER,

APPLIED TO DRIVING MACHINERY.

Sec. 299. The elementary laws of water-power are explained and illustrated in sections 136—140. The student must attentively review those five sections, when he is about to be exercised in the present application of this power. I shall go no farther into the subject, than is necessary for preparing the student for those duties which strictly belong to the *out-of-doors* engineer. I mean, that he must be qualified to take the original measurements, and calculate the power of any proposed mill-seat, before the commencement of any of the works. In doing this, he takes flouring mills as his standard ; estimating their powers by the quantity to be floured in a

given time. Then it is the business of the *mill-wright* and *machinist** to make comparisons, and construct the works according to circumstances.

SEC. 300. The first step to be taken is, to take the necessary measures, and to calculate in cubic feet, pounds, or tons, the water which flows by any point in the stream per second. Directions for this operation are given under the head of Measurements of Excavations and Embankments, sec. 221 and 222. But it may be well to give more particular directions here.

SEC. 301. To find the supply of water, select a time for taking measure when the stream is at its lowest, highest, or middle state, according to the object of the owner. Sometimes a mere flood-mill is desired—in other cases it is not desired as a drought-mill, &c. Select a trunk of the stream which is the most uniform in width, depth, and velocity, and traverse one shore with the compass. Its length ought to be such, that sticks, leaves, &c., will require at least 10 seconds to flow through it—20 seconds will be better, if such a trunk can be found, that is nearly uniform in width, depth and velocity. The length being taken in feet proceed as follows.

SEC. 302. Take measures for a transverse area at every material variation in depth, width, or direction, in this manner: Measure the breadth, and also the depth, at every material difference in depth—be particular to notice the distances between the places where depths are taken. Also measure the distances between the measured areas—all in feet and decimals of feet.

SEC. 303. Take the velocity of the stream in this manner: Suspend a pendulum for beating seconds 39.1 inches in length—or use a watch with a second hand. The pendulum is preferable. Let a careful assistant note the seconds. If the water is shallow, throw in branching weeds, green bushes, &c., of such forms that they will be driven along by the action of the stream from the top to near the bottom.† Note the seconds they occupy in running through, about eight or ten times, in the strength of the current. Then try the same experiment as many times towards each shore; and use your judgment in estimating the proportional part of water in the trunk

* Oliver Evans, and his editors since his decease, have prepared a work, under the title of Mill-wright's Guide, which surpasses all commendation. It is a remarkable specimen of the union of science and art.

† Shavings of white wax are best.

where the side experiments were tried—always bearing in mind that the diminution of velocity as well as of the depth, are to be taken into view; for though the diminution of depth will come into the calculation of areas, its retardation by adhesion at the shallow bottom and shore, must be separately estimated, as near as may be.

Note. These being all the measures to be taken in the field, you will return and make the calculations.

Sec. 304. A plot of the following kind, will greatly facilitate the operation. Fix on the scale by which your plot shall be made; then draw two horizontal lines at a distance from each other equal to the length of the trunk. Plot the traversed shore of the trunk, which will terminate in the parallel lines. Then lay off all the transverse sections, parallel to the horizontal lines, and the true distance from each other. Consider these lines as those drawn across the straight surface of the stream. Let fall perpendiculars from each, according to depth of the measures taken. Connect the lower ends of these measures, which will exhibit each transverse area. These areas may then be calculated by Lapham's method, as described in sec. 225, or they may be cut up into triangles and trapezoids, as described under land surveying, sec. 94 and 95.

Sec. 305. The cubic contents of each section of the trunk may then be cast, by adding the areas of the ends to four times the area of the middle, and multiplying that sum by one sixth of the length. (See sec. 222.)

Sec. 306. Having found the cubic contents of the trunk in cubic feet, average the number of seconds, which the branches, &c., occupy in passing through the length of the trunk. Divide the contents by the seconds, and the quotient will be the cubic feet which pass by the lower point in the trunk per second. Cubic feet may be reduced to pounds by multiplying by 60, and pounds into tons by dividing by 2000. Thus you will have the cubic feet, the pounds, and the tons, which the stream supplies every second.

Note. Here, as in other cases, I adopt the ton of the revised laws of the State of New-York. Students have only to apply common sense, when they have occasion to adopt the gross ton (2240) and, consequently, corresponding numbers in other weights.

Sec. 307. It frequently happens, that a dam is built, and works are already in operation; but the whole of the water is not employ-

ed. Additional works are to be added; and the engineer is called upon to estimate the quantity of waste-water, which pitches over the dam. The word WEIR, or WASTE-WEIR, is applied to this water-pitch, because such a pitch is in use for passing off the excess of water, in freshets, from canals, &c. It takes its name from the strong *wires* (weirs in German) which are inserted, to prevent injuries, which might occur by drifting over small articles of value. Directions for calculating the quantity of water in cubic feet, which crosses the weir per minute, are given in the next section.

SEC. 308. Take the depth of the sheet of water by setting a very thin scale with a sharp edge against the stream, just touching the extreme edge of the waste-board. This measure will be sufficient if the said edge is perfectly horizontal throughout the width of the sheet of water. But the sheet must be divided into sections, and they measured separately, for each change in its level. Take the measure of the whole breadth of the dam, as well as of each section which you may think it necesary to make. Take the cubic feet per minute, set in the table against the inch of depth. This would be the true answer required, if the sheet of water was but one inch wide and confined by side-boards or walls. But if the sheet of water is 50 feet wide, or of any other width more than an inch, multiply the said cubic feet which pass down in the inch sheet per minute, by the whole width of the sheet, taken in inches. This would give the required answer, were the flow of water the same in a confined situation, as when flowing freely over a broad space. To compensate for this difference, divide the above product by 20, and add the quotient; which will give the true answer.

SEC. 309. If the measure of the depth is found in inches and quarters of inches, take the whole in quarters. As for 2 inches and 3 quarters, look for the cubic feet against 11 inches (as this is the number of quarter inches) and take the eighth part of the said cubic feet, set against 11 inches.

SEC. 310. If the table of depths is not sufficient for the measure of the depth of the sheet, take such one of the depths as will produce the measure by doubling, tripling, quadrupling, &c., and multiply the cubic feet set against the taken depth, by the formula set against said doubling, tripling, &c., so assumed. Then divide said product by 20, and add the quotient, as in other cases. By this me-

thod the largest rivers, which fall down perpendicular rocks, may be calculated: and it is the best method in all water pitches.

Sec. 311. *Table for calculating Weirs, or sheets of water falling over dams, over falls, or upon over-shot wheels.*

Depth of the sheet of water to be calculated; taken precisely at the edge of the plunge.	Cubic feet of water per minute; discharged when the sheet of water is one inch wide.	Number of times for doubling, trippling, &c., as referred to in the last section.	Formula of numbers set against each doubling, tripling, &c. to be used as multipliers, set forth in the last section.
1	0.428	Twice taken.	2.828
2	1.211	Three times.	5.196
3	2.226	Four times.	8.000
4	3.427	Five times.	11.180
5	4.789	Six times.	14.697
6	6.295	Seven times.	18.520
7	7.933	Eight times.	22.627
8	9.692	Nine times.	27.000
9	11.564	Ten times.	31.623
10	13.535		
11	15.632		
12	17.805		
13	20.076		
14	22.437		
15	24.883		
16	27.413		
17	30.024		
18	32.710		

Note. This table and the preceding directions for its use, suppose the pond above the fall to be as nearly stagnant as it can be, when it merely gives motion to the sheet of water. If the water reaches the fall with any material degree of velocity, proceed thus: 1st. Calculate the area in feet of the water-sheet at the pitch, by multiplying its depth by its width. 2d. Find the distance in feet which the water above the dam flows per minute, in the usual way, by throwing in floating bodies. 3d. Multiply said distance by said area; and add the product to the cubic feet obtained by the application of the table, as before directed. This sum will be the cubic

feet of waste-water which pitches over the dam, or weir-board, per minute.

Sec. 312. *The descent of water to a lower level* is all that is to be estimated in calculating its power. After the supply of water afforded by a stream is estimated, its power in driving machinery is to be calculated by applying the laws of Hydrodynamics, as set forth in sections 136—140, referred to in section 299 of this article.

Sec. 313. *Directions for calculatiug the efficiency of water-power as issuing from the side of the bottom of a flume.* Measure in feet and decimals, the height of the point intended for the uniform level at the top of the flume, above the central point of the place intended for the gate-hole. Extract the square root of that measure. Multiply 8 (the length of the horizontal jet per second under 1 foot head) by the said root; which product will be the length of the jet of water per second at said gate-hole—its velocity to be estimated at the precise end of the contraction of the vein; about the reduction of one third. Find the area of the intended gate-hole in feet and decimals; and (after deducting for contraction of vein) multiply it by the length of the jet; which will give the cubic feet issuing per second. This will furnish the true measure of the water to be taken per second from the calculated supply of the stream.

Sec. 314. Each cubic foot of water, taken at the top of the stream, weighs 60 ℔. But when it issues as a jet, or spouting fluid, from under a considerable head, it strikes with a force far exceeding its mere steelyard weight. Take this example for an illustration of the principle explained in section 140. Falling bodies, as a cubic foot of ice weighing 60 ℔., increase their velocity as the square root of the distance fallen. Beginning with 60 ℔. (a cubic foot ice-cake) and suppose it merely pressing with 60 ℔. weight. In falling (suppose in a vacuum) its acquired velocity is 8 feet per second at the end of one foot. In falling 9 feet its acquired velocity is 24 feet per second; striking with the weight of 1440 ℔. per second, if such blocks follow each other instantaneously. Its velocity gives an increased impetus against whatever it strikes, according to the increased velocity of each block. As the steelyard weight is merely 60 ℔. pressure, without any acquired velocity, its impetus will be as the square of the acquired velocity.

Sec. 315. In turning an undershot wheel, without any load, the

periphery of the wheel will move just as fast as the water. For example; as water, issuing from a flume of 16 feet head, will move with a velocity of 32 feet per second (see sec. 140) a wheel of 10 feet diameter will perform a revolution every second, and a small fraction over. The velocity which a given head will produce with the periphery of a wheel, where no friction nor power required to move machinery are allowed for, can be calculated in a very simple manner. Find the velocity of the given head by section 140. Calculate the quantity of water, in cubic feet, effused at the gate-hole per second, after allowing for the contraction of the vein. Reduce the cubic feet to pounds. Then you have the velocity of the rim of the wheel, and of its effective power in pounds, for each second; but nothing is yet allowed for friction or load. This must depend solely on careful trials.

Sec. 316. The improvements made in machinery, since the time that Evans made his trials and calculations, are so great, that I believe I shall proceed best in the object of this treatise, by making some calculations upon facts learned at the Poestenkill mills in this city (Troy, New-York.) I shall leave the comparison, as to efficiency, between undershot, breast, and overshot wheels, to millwrights. It is my opinion from my own observations (but they have not been extensive; and I do not offer my opinion as a shadow of authority on this head,) that the horizontal submersed wheels are the best of the undershot kind. I shall treat the subject as if the three modes were equal, if the workmanship is equal.

Sec. 317. *Standard of the efficiency of water-power, taken from Poestenkill flouring mills, in Troy, New-York.*

Three flouring mills on Poestenkill, called Troy mill, Canal mill, and Globe mill, have been recently rebuilt upon the most approved model for overshot mills. The water-wheels of the Troy mill and Canal mill are 18 feet in diameter—with buckets $17\frac{1}{2}$ feet long; and the water pitches upon each from but one foot above. Each turns four run of stones. These are the property of Mr. Phineas H. Buckley. The Globe mill has its water-wheel 12 feet in diameter, with buckets 20 feet long; and the water pitches upon the wheel from about two feet above. This wheel turns five run of stones. It is the property of Messrs. Vail and Townsend. The

three mills are near each other, and are driven by the same water, taken in succession.

Sec. 318. The Troy mill and the Canal mill are constructed so nearly similar, that the description of either applies to both. This day, Feb. 13, 1838, I examined one of the mills, assisted by W. Lapham, a student; and received full explanations of all we desired from the proprietor and his experienced miller. When the water is at a middling height, if but two run of stones are driven, each grinds 250 bushels of wheat in 24 hours—that is, a little more than 10 bushels per hour. By careful measure and two calculations, we find that 40 tons of water strike the wheel per minute, or two thirds of a ton per second. The wheel revolves four times per minute, and each of the stones, which are 4 feet 9 inches in diameter, revolves 146 times per minute. Either two run of either mill, grind 500 bushels in 24 hours; but when the whole 4 run are in action in either mill they will all grind but 700 or 800.

Sec. 319. The Globe mill, with its 12 feet wheel and 20 feet buckets, grind 250 bushels in 24 hours with one run; but I understood, that it could scarcely grind 500 bushels with two run at once. But that the whole five run would grind about as much in 24 hours, as the four run in either of the other mills.

I shall not describe the geering any farther, than to say, that they have no primary cog-wheels on the water-wheel shafts. Cogs are set upon the end of the perimeters of the water-wheels, and mesh into small cog-wheels on transferring shafts. These shafts and cog-wheels transfer the power of the water-wheels to the cog-wheels which turn the spindles.

Sec. 320. I will now attempt to make some applications of Evans' rules to the preceding facts. After a calculation has been made by the engineer of the supply of water, and he has taken a measure of the fall, &c., he will compare the efficiency of the mill-seat, by extracting the square root of his water head, and compare it with the square root of the head at these mills. The rule is the same, in the application of the square root of the height of the water, as illustrated in sections 137 to 140; always having in view, *the supply.*

Sec. 321. *Calculations for efficiency and supply of water.* Evans says, that for grinding 3.8 bushels per hour, 36.582 square feet of the face of the stones must pass over each other per minute. In

pursuance of his mode of estimating, I have made the following calculations. The dressed faces of the stones of the Poestenkill mills are 17.278 square feet to each. Therefore, at every revolution of the stones, 298.5 square feet of them rub over each other. As the stones revolve 146 times in a minute, it follows that 43.585 square feet receive the triturating rub per minute. And 10 bushels are ground per hour ; therefore something more than two million and a half square feet of mill-stone face (2.615.100) are applied to grind 10 bushels. And as 40 tons of water are expended upon the wheel per minute while driving two run of stones in grinding 10 bushels each per hour, this is the clear result : 2400 tons of water, driving stones to rub each other's faces over surfaces of about five and a quarter million feet (5.230.200) will grind 20 bushels of wheat in one hour, with two run of stones, driven by one wheel.

Sec. 322. If the Poestenkill mills may be taken as standards of reference, we may apply these abridged data : 40 tons of water applied per minute to an 18 feet wheel with 17½ feet buckets, will drive two run of stones to triturate about five million square feet of faces in grinding 20 bushels of wheat, *in one hour*. From these data smaller or larger streams and fallsmay be estimated ; always having reference to the proportional principle of square root, as explained in sections 137 to 140.

Sec. 323. To understand this example for calculating the supply of water on the weir method, when pitching from the termination of an apron, or over a weir-board, see sec. 307 to 311.

The first example was calculated from measurements taken of the depth of the sheet of water pitching upon the wheel, depth 5½ inches, and width 17½ feet. As 5.5 inches have no formula in the table, 22 quarters must be taken, which is beyond the table. There-

fore the 22 quarters must be made out according to the rule given in sections 309, 310. This gives the

5.652162	as the cubic feet for 1 inch breadth.
209	width of the sheet in inches.
50869458	
113043240	
20)1181.301858	
59.065	the twentieth to be added.
1240.366	cubic feet.
60	pounds in weight per cubic foot.
2000)74421.960(37.21	tons per minute.
6000	
14421	
14000	
4219	
4000	
2196	
2000	
196	

Sec. 324. For several reasons, I was not satisfied that $5\frac{1}{2}$ inches was the true depth of the sheet; therefore the following calculation was made, on the assumption that the sheet of water was 6 inches deep and $17\frac{1}{2}$ feet wide—or nearly so.

According to the table, 6 inches depth gives, in a sheet of 1 inch width, 6.295 cubic inches per minute.

```
                 6.295  cubic feet for an inch in width.
                   209  width of the sheet in inches.
                 -----
                 56655
               125900
               -------
           20)1315.655
                65.782  twentieth to be added.
              --------
              1381.437  cubic feet.
                    60  pounds weight of cubic feet.
            ----------
2000)82886.220(41.44  tons per minute.
     8000      37.21  tons per minute, sec. 323.
     ----      -----
     2886       4.23  difference.
     2000
     ----
      8862              40 tons per minute the best average
      8000                 for all the wheels.
      ----
       8622
       8000
       ----
        622
```

Sec. 325. *Examples of a calculation of the triturating surfaces of mill-stones.* The stones of Poestenkill mills are 4 feet 9 inches in diameter, a nine-inch space in the centre is allowed for the spindle, &c., unfaced. The mean circle is 8.639 feet. The breadth, outside of the nine-inch area of the centre, is 2 feet. The 2 feet multiplied into the mean circle (8.639) gives 17.278 as the area of the triturating superficies of the stone.

Sec. 326. The triturating superficies of the stone is to be multiplied by itself; for every hair-breadth starting of the stone gives friction for all its superficies. Therefore 17.278 is to be multiplied by itself, to give the superficial friction of one revolution. It must then be multiplied by its revolution per minute. In this case there are 146 revolutions per minute. Therefore the product of 17.278

multipled by itself (producing 298.5) must be multiplied by 146. This gives 43585 per minute.

Example of a calculation for the surface of collision which is necessary between mill-stones, for grinding 20 bushels per hour.

The faced surface of each mill-stone, used in the Poestenkill mills, being 17.278 feet, it must be multiplied by 17.278, to produce the superficial measure of surface in the act of friction. As this multiplication gives the surface of friction, or trituration, at but one revolution, this product must be multiplied by the number of revolutions as aforesaid, per minute (146 times in this case.) The operation is, therefore, as follows:

17.278	the superficial area of the stone.
17.278	
138224	
120946	
34556	
120946	
17278	
298.528284	surface of trituration for one revolution.
146	revolutions per minute.
1791168	[reject three last decimals.]
1194112	
298528	
43585.088	area of the trituration per minute.
60	minutes per hour.
2.615.105.28	area of trituration per hour in grinding 10 bushels on one run of stones.
Or, 5.230.210.56	area for 20 bushels, by this force of water, on two run of stones, driven by one wheel.

TOPOGRAPHY.

SEC. 327. Topography* is the science and art of locating definitely, and describing accurately, any district or limited portion of

* Topography (*topos*, place, *graphe*, description, Greek) is applied, *literally*, to any spot of earth, great or small; but it is mostly applied to the description of a large district of country.

territory, for the purpose of effecting a particular object. It may be applied to the object of locating a fortification—of locating a proposed city—of locating (on a map) rivers, mountains, rock-strata, coal beds, marble quarries, &c. This article will be confined to the mathematical operations required in all cases of accurate topography. Students in engineering must be exercised in a kind of miniature series of operations; and this will be sufficient to qualify an ordinary genius for extensive undertakings. The principle being the same, enlarging a plan will not embarrass the correct scholar, who has carefully gone through a small-scale process.

Sec. 328. After studying, *efficiently*, the preceding rules and directions, nothing remains but additional applications. A case in practice, historically given, is preferred to series of rules. Such cases are within the limit of any school.

The students of Rensselaer Institute undertook (several classes in succession, during the last four years) to settle the relative topography of Durham Peak on Catskill mountain, the Rensselaer Institute in Troy, and the Wiswall house on the west side of the Hudson river. The first step (*extemporaneous step*) was to settle the proximating height, course, distance, &c., by the barometer, sextant, and compass. They used the barometer at each station as described in sec. 168—the sextant as described under instruments, and under latitude and longitude—the compass as described under surveying. The results of these observations they plotted, as a guide for accurate observations; taking the differences of latitude and longitude for measured lines.

Sec. 329. They commenced the series of *definite observations* by assuming a suitable location for a base line—they took Fourth-street, in Troy, because it was accurately levelled about a mile in length. The height of the street above the tide-water level of the Hudson river was taken. At each end of the measure on Fourth-street, the bearings were taken by the compass, to the Wiswall house and to the Rensselaer Institute. The angular distances were also taken with the sextant, from monuments on each end of Fourth-street. It must here be remarked, that the slits in the sights of the compass gave the angles at the Institute and Wiswall's on the level with Fourth-street; but that the sextant gave the angles on the principle of an oblique plane to them from Fourth-street—of course the latter

angles would be smaller than the former, and proportionably larger on the base line. Common sense will suggest the results of calculations on each.

Sec. 330. The length of the base line from the Institute to Wiswall's, could be obtained without further direction, by any student, who had studied this treatise so far, with attention. Having this base line, the distance to Durham Peak was readily obtained. The bearing of Catskill mountain from the Institute and from Wiswall's, was taken by the compass, and the angles in both places by the sextant. The level to the mountain was also taken at both places, and the angular height above the levelled point on the mountain. The height was calculated by the base line to the mountain, and the angle. The descent to tide-water level, below the point of levelling, was taken by calculating the convexity of the earth.

Sec. 331. The height of the Institute and of Wiswall's above tide-water were taken by the level and rod; the barometer being considered but an approximating instrument, well adapted to an extemporaneous survey. With these data, any student in the preceding part of this treatise, can make out all that is required. And he will be able to use the distance from Wiswall's to Durham Peak, as a base line to find the distance to Saddle mountain, near Williams College, in Massachusetts. Then he could use the distance from Saddle mountain to Catskill mountain for a base line to find the distance to Beacon mountain in the Highlands on Hudson river. Thus he might go on indefinitely, and settle the topography where-ever he chose. But, as errors of small beginning will increase in a fearful ratio, a base line of one mile cannot be relied on for such extensive measures.

Sec. 332. Having illustrated the principle sufficiently for the comprehension of any correct student, I will now extend the same method to settling the topography of whole States or of any other territory of ever so great extent. More exact instruments are always required in extensive surveys; as errors will increase by extending lines, which are erroneous at the outset in a small degree. A theodolite, or other *telescopic* levels and graduated instruments, must be employed in all cases, where the reflecting quadrant or sextant are not used. For no ordinary sighting instrument is sufficiently accurate. I say I will extend the method; but I do not

mean to extend my *directions* any farther than to say, that all the preceding directions require enlarged measurements, greater care, frequent reviews, and telescopic instruments.

Sec. 333. For extensive topographical surveys, several base lines of great extent ought to be established. Take for example, the State of New-York. A base line might be surveyed on the ice in the Hudson river, of 50 or 60 miles in length; and monuments erected on shore for each mile, carefully noting the precise distance at right angles from the place of the line on the ice. Other base lines might be established along the shores of frozen lakes, and in such extensive plains as Schoharie Flatts, the 70 mile canal level, &c., &c. These might be referred to for correcting trigonometrical surveys deduced from different bases.

Sec. 334. In taking a topographical survey, much more depends on the ingenuity and faithfulness of the surveyor, than on any parade of outfit, or train of subordinates and assistants. One good *theodolite* of the best modern structure,. a good *sextant*, a *telescopic level*, a *sea-telescope* for taking longitude by Jupiter's satellites, and a well-tried *pocket-chronometer*, will be sufficient for a practical mathematician, who does not make his employment "a sinecure." But faithful and intelligent flag-men, rod-men, and chain or tape-bearers, I have always found to be of more importance than a host of assistants, crowded into the corps to gratify influential relatives.

MATERIALS FOR CONSTRUCTION.

Sec. 335. The world is full of books on this subject. It is rather a *reading* subject than a mathematical one. Any one can sit in his closet and write (rather compile) a treatise on materials for construction. Though this treatise is prepared *solely* for practice, materials for construction in general come within its object. For a reading book, Professor Mahan's treatise is recommended as a compend of every thing worth reading on that subject.* I propose but few materials on this vastly extended subject; and these are intended to be adapted to the discipline of students *in mode of thinking* on the subject, rather than to the theory and practice.

* It is a matter of regret that the title of this excellent treatise on materials for construction, was so inappropriately selected.

Sec. 336. *Materials for construction* are very naturally divided into Inorganic and Organic. The *Inorganic* are those which are governed by the laws of *affinity*, uninfluenced by the living principle—as rocks, earths, and metals. *Organic* are those which received their forms or structure from the unexplained action of the *living principle*—as timber, bones, teeth, &c. As they owe their origin to a forced structure, induced by the living principle, they are predisposed to a departure from their present structure; consequently are less durable.

Inorganic Materials for Construction.

Sec. 337. These are subject to decay on two principles—*decomposition* and *disintegration*. Decomposition is the separation of constituent atoms, by the interference of extraneous atoms which, by force of chemical attraction, draw asunder previously combined atoms. Carbonic acid, for example, unites with atoms of iron, separating them from the general mass, and forming the yellowish iron-rust. Oxygen also unites with atoms of iron, forming the red iron-rust. Disintegration separates whole molecules of masses, without separating their constituent atoms; as the crumbling down of a rock of slate, of limestone, of graywacke, &c. In such cases the smallest fragment, or molecule, is but a lessened rock, retaining its original atomic constituents.

Sec. 338. Most persons know the names of *timber*, as oak, pine, cedar, &c., at sight; and need no direction on that subject. Few know the names of *rocks* and *earths;* and need some instruction on the characteristics by which they are known. As they are pretty accurately distributed into strata by modern geologists, I shall give a few concise directions by which they are known. I consider this the more important, as canal and rail-road reports recently include geological characters.

Geological Alphabet.

Sec. 339. Rocks are mostly made up of aggregations of homogeneous minerals; in some cases a rock consists wholly of a single homogeneons mineral—as limestone, argillite, &c. The annexed *Geological Alphabet* must be learned by an inspection of the specimens *only;* no description can convey an adequate idea of them.

Sec. 340. Every stratum of rock, or earth, consists of one or more of these nine minerals; therefore they are aptly denominated the Geological Alphabet. They are—1. Quartz, 2. Felspar, 3. Mica, 4. Talc, 5. Hornblende, 6. Argillite, 7. Limestone, 8. Gypsum, 9. Chlorite.

Quartz. When held between the eye and a window, it reflects light somewhat like a polished piece of cold tallow or glass. This is called its lustre. On attempting to scratch it with a pen-knife, the metal will leave a trace on it, and it will not be scratched. It is commonly glass color, but it is often milk-white, reddish, and of various other colors. Quartz consists of about 93 per cent silex, 6 alumin or clay, 1 lime, besides 2 or 3 per cent of water in a solid state.

Felspar, or *Feldspar.* Its lustre is peculiar; but it in some measure resembles that of a broken edge of china-ware. It may be scratched with a knife. Its color is generally white or flesh-colored. It is best ascertained when in the state of a rock aggregate, by procuring an outside fragment, which had been long exposed to air and moisture. In this state it always assumes a peculiar tarnish, of a dirty yellowish hue. Felspar consists of about 63 per cent silex, 17 alumin, 13 potash, 3 lime, 1 iron, 3 water.

Mica. It is always in shining lamina or scales. It is every where known by the improper name of isinglass. The scales are always elastic. It consists of 48 per cent silex, 34 alumin, 9 potash, 5 iron, 1 manganese, 3 water. Black mica contains 22 per cent iron.

Talc. It often resembles mica; but can be distinguished from it by being non-elastic. Take a small scale or fibre of it in a pair of tweezers, put it under a magnifier, and bend it with the point of a fine needle. If it remains as it is bent, it is talc; if it springs back it is mica. It always gives a rock the unctuous or soapy feel. It consists of 62 per cent silex, 27 magnesia, 2 alumin, 3 iron, 6 water.

Hornblende. It is the toughest of all earthy minerals. Generally it presents a kind of confused fibrous structure. It may be scratched with the knife. The color is always greenish, brownish, or black. Sometimes it appears in black scales, resembling mica to the naked eye; but under the magnifier it differs materially. It

consists of about 42 per cent silex, 12 alumin, 11 lime, 3 magnesia, 30 iron, 1 manganese, 1 water. It is very heavy.

Argillite. This needs no description. The common roof-slate, and the slate used for cyphering on in schools, are good specimens. It consists of about 38 per cent silex, 26 alumin, 8 magnesia, 4 lime, 14 iron, 10 of potash, soda, manganese, water, &c.

Limestone. Common marble affords good specimens. It should be tested by a drop of muriatic, nitric, or sulphuric acid, which will cause an effervescence or bubbling. It consists of 57 per cent of lime, 43 of carbonic acid. Sometimes it is colored with iron; and often contains a little silex and alumin.

Gypsum. Common plaster of Paris. It will not effervesce with acids, and is generally softer than limestone. It consists of 32 per cent lime, 46 sulphuric acid, 22 water.

Chlorite. It is a little harder than talc, but may be scratched with the finger nail. Under the magnifier it appears like a compact mass of fine green scales. When breathed on it gives an odour in some measure resembling clay. Its elementary constituents are variable. They will average about 40 per cent silex, 23 alumin, 18 magnesia, 15 oxid of iron, 2 lime, 2 water. This is the least important of the whole nine.

After students can readily recognise these nine minerals, they should be exercised in pointing them out in their various states of aggregation. They will soon be enabled to spell out any rock with facility.

Arrangement of Rocks for the Instruction of Engineers.

Sec. 341. The science of Geology consists in a systematic arrangement of facts, explaining the structure of the earth.

Our observations are limited to its exterior rind or coats. We know very little of its interior structure. But the inequalities of its surface often give us admission to a considerable depth; from which we should be totally excluded were its surface every where smooth like a Pacific sea.

Geology teaches us that minerals which are associated in one district of country, are associated in the same order in all other districts. Hence the experience of the miner and the quarry-man in

any country, may be applied in searching for useful minerals in all other countries; for geology is the true science of mining.

Five classes or series of Deposites.

We have five series of strata, or a five-fold repetition of a carboniferous, quartzose, and calcareous formation.

First series (primitive) is terminated by *granular limerock,* destitute of any organized remains.

Second series (transition) is terminated by *compact* and *shelly limerock;* the compact perforated with encrinites.

Third series (lower secondary) is terminated by *corniferous* or *cherty limerock.* It contains horn-stone and abounds in stone horns.

Fourth series (upper secondary) is terminated by *oolite* or *coral rag,* a calcareous stratum containing corals.

Fifth series (tertiary) is earthy, and terminated by a lime deposit of shell-marl.

To these may be added the *red sandstone group,* or the gypsum and salt associates; as along the Erie canal, between Utica and Lockport. Also the cretaceous deposite; as the green-sand marle of New Jersey, &c.

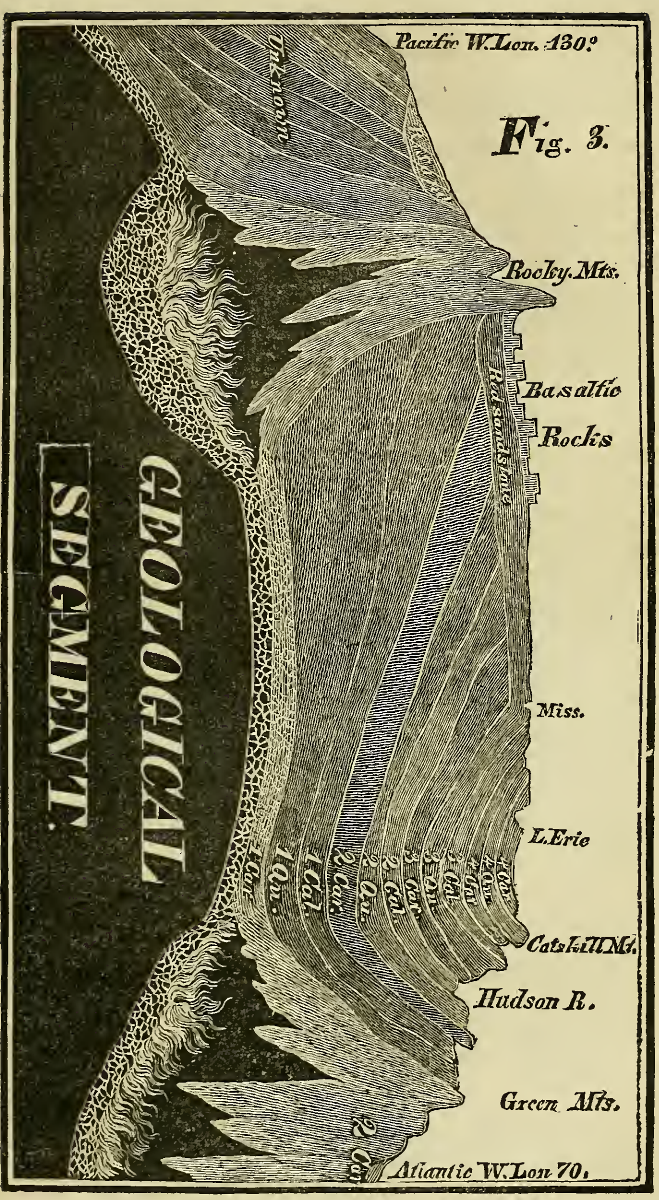
Exhibition of Geological Strata.

[From the Geological Text-Book.]

Sec. 342. *Fig.* 3. This figure represents a segment of the earth, from the Atlantic to the Pacific, between the 42° and 43° N. latitude, in its present state, so far as regards rock strata.

Abbreviations. Car. Carboniferous formations; Qu. Quartzose formations; Cal. Calcareous formations. The numerals indicate the first, second, third, fourth, and fifth series of formations. The fifth, however, is not represented here.

According to the modern theory of the earth, these strata of rocks were deposited in concentric hollow spheres, like the coats of an onion. And within the *granitic sphere,* combustible materials were deposited; and all strata above were broken up in several north and south *rents*, by their combustion and explosion. For example, one passed through New England, the Highlands, Virginia, &c.; another included the Rocky Mountains, Andes, &c., as here exhibited.


Pacific W. Lon. 130°
Fig. 3.
Unknown
Rocky. Mts.
Basaltic
Rocks
Red sandstone
Miss.
L. Erie
Catskill Mt.
Hudson R.
Green Mts.
Atlantic W. Lon 70.
GEOLOGICAL
SEGMENT.

Descriptions of Strata.

Sec. 343. Class I. Primitive or First Series.

1. *Carboniferous or slaty formation.*

Granite, is an aggregate of angular masses of quartz, felspar, and mica. Subdivisions. It is called *crystalline* (granite proper) when the felspar and quartz present an irregular crystalline, not a slaty, form. It is called *slaty* (gneiss) when the mica is so interposed in layers as to present a slaty form.

Mica-Slate, is an aggregate of grains of quartz and scales of mica.

Hornblende Rock, is an aggregate, not basaltic, consisting wholly, or in part, of hornblende and felspar.

Talcose Slate, is an aggregate of grains of quartz and scales of mica and talc. Subdivisions. *Compact*, having the laminæ so closely united that a transverse section may be wrought with a smooth face. When the quartzose particles are very minute and in a large proportion, it is manufactured into scythe-whetstones, called Quinnebog stones. *Fissile*, when the laminæ separate readily by a blow upon the surface. Varieties. *Chloritic*, when colored green by chlorite. It contains gold in the Carolinas, and probably throughout its whole range by way of New-York, to Canada.

2. *Quartzose formation.*

Granular Quartz, consists of grains of quartz united without cement.

3. *Calcareous formation.*

Granular Limestone, consists of glimmering grains of carbonate of lime united without cement. *Dolomite*, when it consists in part of magnesia, and is friable.

Sec. 344. Class II. Transition or Second Series.

1. *Carboniferous or slaty formation.*

Argillite, is a slate rock of an aluminous character, and nearly homogeneous, always consisting of tables or laminæ whose direction

forms a large angle with the general direction of the rock. *Clay Slate,* when the argillite is nearly destitute of all grittiness, and contians no scales of mica or talc. *Wacke Slate,* when it is somewhat gritty and contains glimmering scales of mica or talc. *Roof Slate,* when the slate is susceptible of division into pieces suitable for roofing houses, and for cyphering slate.

2. *Quartzose formation.*

First Graywacke, is an aggregate of angular grains of quartzose sand, united by an argillaceous cement, apparently disintegrated clay slate, spangled with glimmering scales. *Millstone grit and grey rubble,** when the grains are in part coarse, and more or less conglomerate, either white or grey, often very hard.

3. *Calcareous formation.*

Sparry Limerock, consists of carbonate of lime, intermediate in texture between granular and compact; and is traversed by veins of calcareous spar.

Calciferous Sandrock, consists of fine grains of quartzose sand and of carbonate of lime, united without cement, or with an exceeding small proportion.

Metalliferous Limerock, consists of carbonate of lime in a homogeneous state, or in the state of petrifications. *Birdseye marble,* when the natural layers are pierced transversely with cylindric petrifactions, so as to give the birdseye appearance when polished.

Sec. 345. Class III. Lower Secondary or Third Series.

1. *Carboniferous or slaty formation.*

Second Graywacke, is an aggregate of grains of quartzose sand, less angular than those of first graywacke, and generally contains some fine grains of limestone.

It is sometimes gritty, and contains a few glimmering scales; but it is often a soft slate and dark brown. It rests upon the

* *Rubble* being an uncouth word, but too well established to be rejected, I will state: that in common English it signifies a hard grey stone, in roads, of a spheroidal form, which causes the rumbling and jolting of carriages. Kirwan calls these stones, common graywacke, as opposed to graywacke slate.

shelly kind of transition limerock, and is the lowest of our secondary strata.

2. *Quartzose formation.*

Millstone Grit and Rubble, are composed of quartzose pebbles and grains cemented together; often very hard.

Remark. A distinct stratum called *old red sandstone*, and another called *millstone grit*, have been given by most geologists. But the latest European geologists very properly reject them. Because the red sandstone is found passing into all the three graywackes.

3. *Calcareous formation.*

Geodiferous Limerock, consists of carbonate of lime, combined with a small proportion of argillite or quartz in a compact state, mostly fetid, and always containing numerous geodes.

Corniferous Limerock, consists of carbonate of lime, embracing hornstone, and numerous species of petrifactions, called stonehorns (Cyathopyllum.) This stratum is called carboniferous limestone by Conybeare.

Sec. 346. Class IV. Upper Secondary or Fourth Series.

1. *Carboniferous or slaty formation.*

Third Graywacke, is an aggregate of grains of quartzose sand and pebbles, less angular than those of first and second graywackes, and generally contains fine grains of limestone.

It is sometimes gritty; but often soft. It rests on the carboniferous limerock of foreign geologists, who often call it grit slate.

2. *Quartzose formation.*

Millstone Grit and Rubble, are composed of quartzose pebbles and grains, cemented together; often very hard.

3. *Calcareous formation.*

Oolitic Rocks, are aggregates, which contain more or less of carbonate of lime of an earthy texture, either compact, or granulated.

SEC. 347. CLASS V. TERTIARY OR FIFTH SERIES.

1. *Carboniferous formation.*

PLASTIC CLAY, that kind of clay, generally called potter-baker's clay, which will not effervesce with acids. When it is white it is called pipe-clay.

MARLY CLAY, that kind of clay which will effervesce with strong acids. This stratum is almost universal.

MARINE SAND AND CRAG. The *sand* consists of fine grains of quartz, not united by adhesion or cement, but in loose masses which may mostly be poured. The *crag* consists of pebbles, clay and loam, either united by carbonate of lime or iron cement, as *pudding-stone;* by clay and iron cement, as the *hard-pan;* or not united, being merely *stratified gravel;* or united by adhesion, as the *arenaceous concretions* near Troy, on Green Island. The *marine sand* occupies a broad strip on the west side of Hudson river, from near Lake Champlain to a distance of 100 miles.

SHELL-MARL, is in insulated or continued layers, fields, or patches, in almost every part of the earth. It consists chiefly of broken, pulverized, and entire shells, of the genus *helix* (genera *helix, planorbis* and *lymnea* of Lamarck.)

SEC. 348. SUBORDINATE SERIES EMBRACED IN THE THIRD REGULAR SERIES.

(Lower Secondary.)

1. *Carboniferous and quartzose formation.*

SALIFEROUS ROCK, consists of red, or bluish-grey, sand or clay-marl, or both. In some localities they form the floor of salt mines and salt springs.

2. *Quartzose and slaty formations.*

FERRIFEROUS ROCK, is a soft, slaty, argillaceous, or a hard, sandy, siliceous rock, embracing red argillaceous iron ore.

3. *Calcareous formation.*

LIASOID, is an argillaceous limestone, with an admixture of magnesia, iron, and finely pulverized quartz; forming a compound of

homogeneous aspect. On burning and mixing as in the manufacture of mason's mortar, it becomes a solid cement under water.

Sec. 349. *Anomalous Deposits.*

1. *Volcanic.*

Basalt, which is called *Amygdaloid*, when amorphous, of a close texture, but containing cellules, empty or filled. When the amygdaloid has a warty appearance and resembles slag, it is called *toad-stone*. *Columnar basalt*, is presented in prismatic polygons more or less regular.

Sec. 350. 1. Useful Rocks.

Marble. This name is indefinitely applied to any rock of carbonate of lime, which may be wrought into building stone with the chisel. The most valuable is the Granular limerock. In this country it extends from Canada to Pennsylvania, along the west side of the Green Mountain range. Therefore, every farmer, whose lands lie in that range, may probably find on his farm at a greater or less depth, or at the surface, good statuary marble. Good marble has been found, called birdseye marble, in the compact transition limerocks; also in the shelly kind. But shelly marble will readily disintegrate on exposure to heat; and in time, on exposure to the disintegrating agents.

Freestone. This name has usually been applied to red sandstone. But its application to all saliferous rocks (red, gray, or variegated) is authorised. The freestone of Rochester, on Genessee river, is red, gray, spotted, and variously colored. Saliferous rocks, and red wacke of the first, second, and third graywackes, make good freestone, unless they contain pyrites. These stones never crack by heat; but they crumble off or become friable, and ought not to be highly heated.

Flagging-stones. The best known in this country are the gneiss, and gneisseoid hornblende. Haddam, Conn., affords excellent specimens. West of Worcester, Mass., towards, and in, Leicester, the rocks are equally good. They were formerly too far inland. Since the canal is made, they may become profitable. When I first heard of this project, I supposed the value of the stock was to be es-

timated by the value of these vast ledges of gneiss rock. Why Yankee ingenuity and perseverance does not reach them, I know not.

Wall-stones. This name is applied to stones which lie fairly in a wall, with or without chiseling or sawing. It is not necessary that they should bear chiseling, provided they can be broken from ledges in regular parallelopipeds for laying up in a dry wall, or for masoning with mortar. Gneiss, gneisseoid hornblende, and the two lower graywackes, afford more or less good wall-stone. First graywacke, when formative, is always a good wall-stone. It has been sawed and used in Troy with success, in the basement story of houses. See Dr. Gale's house. It is remarkable for cracking, and even flying into pieces, when heated; hence it is called *snap-stone* in some districts. Third graywacke is rarely suitable for a wall-stone; for it contains iron pyrites, which often produces rapid disintegration. See some of the western locks, and the walls of some houses in Ithaca.

Sec. 351. *Millstone* is a subdivision of graywacke. It is therefore first, second, and third. It is a good wall-stone, and will resist a great heat. Millstones were quarried from Shawangunk Mt. (first gray-wack) until the burrstone manufactories were extensively introduced; and it was profitable to the manufacturers, and a public benefit. As they were wrought at Esopus (Kingston) in great quantities, they were long called Esopus millstones. Ledges of millstone grit, which would make tolerably good millstones, though liable to crumble, may be had in second graywacke near Utica, and in third graywacke on Alleghany mountains.

Grindstone used in this country is a gray sandy variety of third graywacke. Two extensive layers of grindstone run nearly parallel to Schoharie Kill, in Blenheim, Schoharie county, on the estate of Judge Sutherland. One range is in the west bank of the Kill, the other is further west. From pieces of rock adhering to Nova Scotia grindstones, I believe they are from the same rock. Numerous other localities of various qualities, are seen in the graywacke of Catskill and Alleghany mountains.

Whetstones, called novaculite, are always a variety of talcose slate. When they are harsh they are called Quinnebog whetstones, or scythe whetstones. When they are fine-grained they are called

Turkey hone. They are wrought in Belchertown, Mass., and at Lake Memphremagog in Vermont. Though these whetstones are always a variety of talcose slate, they are found in this country at the meeting of talcose slate with mica slate or argillite; when with the latter the whetstone is softer than when with the former. In Hawley, Mass., an inferior kind appears in connexion with the micaceous iron ore, at the meeting of the talcose slate and mica slate.

Hones of a very excellent quality, are found in first graywacke near a place called the Red-Rockshire, in Columbia county, N. Y., and in Rensselaerville, Albany county, in third graywacke. I have seen layers of the same rock in numerous localities in the third graywacke of Catskill and Alleghany mountain ranges.

SEC. 352. *Hornblende rocks* are the toughest of all rocks; consequently useful in fortifications, to resist the force of a cannonade. They are somewhat durable—those of the basaltic kind are best.

Granular quartz and *granular limestone* are the most durable of all rocks; and often present convenient forms or blocks, for use in building.

Argillite, argillaceous graywacke, saliferous slate, ferriferous slate, conchoidal lias, and *pyritiferous slate,* are subject to rapid disintegration, and are not suitable for building materials. But they exceed all other materials for dams and other works, where the force of water is to be resisted. Hence the dam on the Hudson river in Troy, is made to resist the force of that mighty river by argillite taken from Mt. Olympus.

SEC. 353. CEMENTS made of *lime* and *sand* are most in use where walls are not continually exposed to water. The sand should be used immediately from the pit without drying. But if the original stone was made up of carbonate of lime and silicious sand, before it was burned, it makes a better cement for such exposed walls. This fact is mentioned by Glauber, who wrote two centuries ago. Recently, engineer Canvass White, Esq., has most effectually revived the use of this cement in canal locks, &c. Oxyd of iron or of manganese improves the cement.

Gypsum cement. The gypsum is first ground at the plaster-mill; then heated in a potash-kettle, or other iron vessel, for 24 or 36 hours, to nearly a red heat. It is thus divested of its water of combination, being about 20 or 22 per cent. This dry powder is to be

kept from the air, by being enclosed in casks, until the moment it is to be applied. It is then wet with about its weight of water, for common use—more for nice plaster work—less for pointing and common mason work. A little glue is dissolved in the water, when some time is required for applying it, to prevent its hardening too soon.

This cement is excellent for making a smooth finish, mouldings, &c., for inside walls. But it is not very durable on exposed surfaces. The application must be made under the constant view of this fact: that its volume is a little enlarged after it has been applied, by the farther absorption of water, or atmospheric vapor.

Timber Materials for Construction.

Sec. 354. *Timber has always been used*, and is necessary for certain parts of stone or brick edifices—particularly for beams. And if timber is well selected, and well prepared and arranged, it will endure many centuries. The coffins of the Egyptian mummies, which are three thousand years old, are still perfectly sound.

The white oak (quercus alba) has long been considered as combining solidity and durability in a pre-eminent degree. But it is much better in both respects, for having grown in open ground. And the best part of a tree is between the sap-wood (alburnum) and the heart; as the heart is generally shaky, and the alburnum is not sufficiently solidified. Notwithstanding the durable character of the oak, it will not endure exposure to water, as well as cedar, and some other timber. It should be placed in situations which are secured from rains. It should be well seasoned under cover before it is used. If it is soaked in water before it is seasoned, it will be brittle and soon decay. For the water takes out all the soluble parts; which, if dried in the timber, solidify it.

Oak which grows in warm countries, is found to be the best. The soil should be neither wet or dry, and at about a medium in quality. For if it grows too thriftily or stintedly, it will be less solid. Soil, which contains considerable clay, produces better oak than a loose loamy or sandy soil. Very old or very young trees are not as good as those which are about 100 years old.

Most of the preceding remarks will apply to other timbers. It may be added, no timber should remain in the bark after being

felled. Neither should the sap-wood remain, if the timber is required for long duration.

Sec. 355. *Strong wood posts* will sustain very great weights directly in line with their fibres. But sawed posts, which have crooked grains, will be weakened by having their fibres cut off by the saw. No material can be employed which will retain its original length, through all temperatures and all degrees of humidity, so well as wood.

Wood is very durable when placed in situations from which rain is excluded; and its decay is most rapid when it is alternately wetted and dried at short intervals. Hence the bottom of a vessel, which is always immersed in water, endures much longer than the sides (the parts between wind and water, in seamen's language) which are perpetually subjected to the destructive effects of the sun and water.

Direction of pressure. Carpenters should study to bring the greatest pressure as near the direction of the grains of the timber as possible. If a strong oaken piece of timber, one foot long, is suspended by one end, and has a hook at the other, we can scarcely conceive of a weight fastened to the hook sufficient to break it—its ends, however, must be supposed to be fastened by clamp-like bands.

Depth of Beams. If the depth of a beam is great, its thickness may be small; though a proportional thickness must be adapted to each particular case. All tapering timbers should have their tapering calculated upon the common lever principles. Spokes of carriage wheels, arms of mill wheels, &c., taper towards their extremities upon the lever principle.

Cross-grain timber. No honest carpenter will work in cross-grained timber, where strength is required, without giving his opinion of its insufficiency. In all such cases mere lateral adhesion of fibres gives all the strength. Some kinds of wood, such as the common laurel (*kalmia latifolia*) are almost exceptions to the general rule. Still there is no timber which cannot be easiest divided in the direction of the fibres.

Iron Materials for Construction.

Sec. 356. *Kinds of iron.* Iron is distinguished into three general kinds: *Cast iron*, *wrought iron*, and *steel.* Cast iron contains a pro-

portion of carbon, and is of a brittle granulated structure. By melting iron and stirring it while in fusion, part of the carbon is burned out. Then by hammering or rolling it becomes almost pure, fibrous and tough; and is then called wrought iron. After it is brought to the state of wrought iron, it is converted into steel by heating it in a confined place in contact with charcoal, with which it combines. It will then become hard on heating and plunging into cold water, and its hardness will be proportioned to the degree of heat and cold; for its hardness depends on the suddenness and extent of the diminution of temperature.

Sec. 357. *Application of the kinds of iron.* When great strength is required, or jarring and striking motion, *wrought iron* is preferable to all known materials. Rusting of iron, by its ready union with oxygen and carbonic acid, requires that it should be painted, or defended in some other manner. When nothing but hardness is required, *cast iron* is best. But when great hardness, joined to considerable strength is required, *steel* is preferable. If very great strength is required, bars of steel should be welded upon bars of tough iron, as in making sleigh-shoes.

The smith should receive particular directions, respecting the application of his work, from the engineer. For example: if bolts are to resist great force, by pulling lengthwise of them, (as when holding up the string-pieces of a bridge, which is sustained by upper frame-work) care must be taken to have the heads large and to consist of a part of the bolt itself, so constructed that the heads shall be continuations of the bolts. The whole must be the toughest and softest of iron. Bolts for holding timbers from sliding or moving out of place may be made of iron of less tenacity, unless so situated as to be subject to jarring strokes, as in all moveable carriages.

Inorganic Materials for Construction,*

With their specific gravity (see sec. 135) and weight in pounds, per cubic foot. It is well known that their solidity is indicated by their weight.

Names.	Specific Grav.	Weight in pounds.	Names.	Specific Grav.	Weight in pounds.
Atmospheric Air,	.0012	.075	Limestone, compact,	2.598	162.37
Basalt,	3.000	187.50	Limestone, granular,	2.653	165.81
Brick, common,	2.000	125.00	Lime, quick, stone,	.843	52.68
Brick, red,	2.168	135.50	Marble, Parian,	2.837	177.31
Chalk,	2.657	166.06	Marl, common,	1.600	100.00
Charcoal, birch,	.542	33.87	Mortar, paste 3 s-2 l	1.588	99.25
Charcoal, oak,	.332	20.75	Sand, pit, fine,	1.523	95.18
Charcoal, pine,	.280	17.50	Sand, river,	1.886	117.87
Clay, marley,	1.919	119.93	Slate, argillite,	2.781	173.81
Coal,	1.290	80.62	Wacke, gray,	2.614	163.37
Earth, common,	1.984	124.00	Wacke, grindstone,	2.143	133.93
Granite, common,	2.664	166.50	Sienite,	2.621	163.81
Gravel, common,	1.749	109.32	Tufa, calcareous,	1.217	76.06
Gypsum, common,	2.286	142.87	Water, rain,	1.000	60, N. Y.†

Organic Materials for Construction,

With their specific gravity and weight in pounds, per cubic foot.

Names.	Specific Grav.	Weight in pounds.	Names.	Specific Grav.	Weight in pounds.
Alder, dry,	.555	34.68	Oak, live,	1.216	76.03
Almond-tree,	1.102	68.87	Oak, white,	.908	56.75
Apple-tree,	.793	49.56	Oak, red,	.752	47.00
Ash, dry,	.845	52.81	Pear-tree, dry,	.708	44.25
Beech, partly dry,	.854	53.37	Pine, pitch, dry,	.936	58.50
Birch, dry,	.720	45.00	Pine, white, dry,	.460	28.75
Box, dry,	1.030	64.37	Plane-tree, button-wood	.648	40.50
Cedar,	.753	47.06	Plum-tree,	.785	49.06
Cherry-tree, dry,	.672	42.00	Poplar, black, dry,	.421	26.31
Chestnut, dry,	.606	37.95	Poplar, Lombardy, dry,	.374	24.37
Chestnut, horse, dry,	.596	37.28	Quince-tree,	.705	44.06
Ebony,	1.331	83.00	Sassafras,	.482	30.12
Elm, dry,	.588	36.75	Sycamore,	.645	40.31
Fir, black spruce,	.512	32.00	Tulip-tree, white-wood,	.477	29.81
Hickory,	.929	58.06	Vine, grape,	1.237	77.31
Hornbeam,	.760	47.50	Walnut, black,	.920	57.50
Lignumvitæ,	1.333	83.31	Walnut, butternut, dry,	.616	38.50
Mahogany,	.852	53.30	Willow, green,	.619	38.68
Maple, dry,	.755	47.18	Yew,	.788	48.62

* These items of mineral and vegetable materials were selected from Treadgold's tables.

† 62.5 lb. English, per cubic foot.

TABLES, WROUGHT EXAMPLES, &c.

Segment of the Table of Logarithms of Numbers; to be used according to Hutton in calculating heights of mountains, &c., by the barometer.

No.	0	1	2	3	4	5	6	7	8	9
255	406540	406710	406881	407051	407221	407391	407561	407731	407900	408070
256	408240	408410	408579	408749	408918	409087	409257	409426	409595	409764
257	409933	410102	410271	410440	410608	410777	410946	411114	411283	411451
258	411620	411788	411956	412124	412292	412460	412628	412796	412964	413132
259	413300	413467	413635	413802	413970	414137	414305	414472	414639	414806
260	414973	415140	415307	415474	415641	415808	415974	416141	416308	416474
261	416640	416807	416973	417139	417306	417472	417638	417804	417970	418135
262	418301	418467	418633	418798	418964	419129	419295	419460	419625	419791
263	419956	420121	420286	420451	420616	420781	420945	421110	421275	421439
264	421604	421768	421933	422097	422261	422426	422590	422754	422918	423082
265	423246	423410	423573	423737	423901	424064	424228	424392	424555	424718
266	424882	425045	425208	425371	425534	425697	425860	426023	426186	426349
267	426511	426674	426836	426999	427161	427324	427486	427648	427811	427973
268	428135	428297	428459	428621	428782	428944	429106	429268	429429	429591
269	429752	429914	430075	430236	430398	430559	430720	430881	431042	431203
270	431364	431525	431685	431846	432007	432167	432328	432488	432649	432809
271	432969	433129	433290	433450	433610	433770	433930	434090	434249	434409
272	434569	434728	434888	435048	435207	435366	435526	435685	435844	436003
273	436163	436322	436481	436640	436798	436957	437116	437275	437433	437592
274	437751	437909	438067	438226	438384	438542	438700	438859	439017	439175
275	439333	439491	439648	439806	439964	440122	440279	440437	440594	440752
276	440909	441066	441224	441381	441538	441695	441852	442009	442166	442323
277	442480	442636	442793	442950	443106	443263	443419	443576	443732	443888
278	444045	444201	444357	444513	444669	444825	444981	445137	445293	445448
279	445604	445760	445915	446071	446226	446382	446537	446692	446848	447003
280	447158	447313	447468	447623	447778	447933	448088	448242	448397	448552
281	448706	448861	449015	449170	449324	449478	449633	449787	449941	450095
282	450249	450403	450557	450711	450865	451018	451172	451326	451479	451633
283	451786	451940	452093	452247	452400	452553	452706	452859	453012	453165
284	453318	453471	453624	453777	453930	454082	454235	454387	454540	454692
285	454845	454997	455149	455302	455454	455606	455758	455910	456062	456214
286	456366	456518	456670	456821	456973	457125	457276	457428	457579	457730
287	457882	458033	458184	458336	458487	458638	458789	458940	459091	459242
288	459392	459543	459694	459845	459995	460146	460296	460447	460597	460747
289	460898	461048	461198	461348	461498	461649	461799	461948	462098	462248
290	462398	462548	462697	462847	462997	463146	463296	463445	463594	463744
291	463893	464042	464191	464340	464489	464639	464787	464936	465085	465234
292	465383	465532	465680	465829	465977	466126	466274	466423	466571	466719
293	466868	467016	467164	467312	467460	467608	467756	467904	468052	468200
294	468347	468495	468643	468790	468938	469085	469234	469380	469527	469675
295	469822	469969	470116	470263	470410	470557	470704	470851	470998	471145
296	471292	471438	471585	471732	471878	472025	472171	472317	472464	472610
297	472756	472903	473049	473195	473341	473487	473633	473779	473925	474070
298	474216	474362	474508	474653	474799	474944	475090	475235	475381	475526
299	475671	475816	475962	476107	476252	476397	476542	476687	476832	476976

300	477121	477266	477411	477555	477700	477844	477989	478133	478278	478422
301	478566	478711	478855	478999	479143	479287	479431	479575	479719	479863
302	480007	480151	480294	480438	480582	480725	480869	481012	481156	481299
303	481443	481586	481729	481872	482016	482159	482302	482445	482588	482731
304	482874	483016	483159	483302	483445	483587	483730	483872	484015	484157
305	484300	484442	484584	484727	484869	485011	485153	485295	485437	485579
306	485721	485863	486005	486147	486289	486430	486572	486714	486855	486997
307	487138	487280	487421	487563	487704	487845	487986	488127	488269	488410
308	488551	488692	488833	488973	489114	489255	489396	489537	489677	489818
309	489958	490099	490239	490380	490520	490661	490801	490941	491081	491222
310	491362	491502	491642	491782	491922	492062	492201	492341	492481	492621
	0	1	2	3	4	5	6	7	8	9

Refraction.

Bowditch says, that the refraction of terrestial objects may be found thus, very nearly correct, when the air is clear.

Calculate the distance of the object; and find the angle subtended by said distance at the centre of the earth—one fourteenth of said angle is the angle of refraction.

Refraction of Heavenly Bodies in Altitude.

App. Alt.	Ref.	App. Alt.	Ref.	App. Alt.	Ref.	App. Alt.	Ref.	App. Alt.	Ref.
D. M.	M. S.	D. M.	M. S.	D. M.	M. S.	D. M.	M. S.	D. M.	M. S.
0. 0	33. 0	1.40	20.18	3.20	13.33	6.30	7.52	9.50	5.20
0. 5	32.11	1.45	19.51	3.25	13.19	6.40	7.41	10. 0	5.15
0.10	31.22	1.50	19.25	3.30	13. 5	6.50	7.31	10.15	5. 8
0.15	30.36	1.55	18.59	3.40	12.39	7. 0	7.21	10.30	5. 0
0.20	29.50	2. 0	18.35	3.50	12.14	7.10	7.12	10.45	4.54
0.25	29. 6	2. 5	18.11	4. 0	11.50	7.20	7. 3	11. 0	4.47
0.30	28.23	2.10	17.48	4.10	11.28	7.30	6.54	11.15	4.41
0.35	27.41	2.15	17.26	4.20	11. 7	7.40	6.46	11.30	4.35
0.40	27. 0	2.20	17. 4	4.30	10.47	7.50	6.38	11.45	4.29
0.45	26.20	2.25	16.44	4.40	10.28	8. 0	6.30	12. 0	4.23
0.50	25.42	2.30	16.23	4.50	10.10	8.10	6.22	12.20	4.16
0.55	25. 5	2.35	16. 4	5. 0	9.53	8.20	6.15	12.40	4. 9
1. 0	24.29	2.40	15.45	5.10	9.37	8.30	6. 8	13. 0	4. 3
1. 5	23.54	2.45	15.27	5.20	9.21	8.40	6. 1	13.20	3.57
1.10	23.20	2.50	15. 9	5.30	9. 7	8.50	5.55	13.40	3.51
1.15	22.47	2.55	14.52	5.40	8.53	9. 0	5.49	14. 0	3.46
1.20	22.15	3. 0	14.35	5.50	8.39	9.10	5.43	14.20	3.40
1.25	21.44	3. 5	14.19	6. 0	8.27	9.20	5.37	14.40	3.35
1.30	21.15	3.10	14. 3	6.10	8.15	9.30	5.31	15. 0	3.30
1.35	20.46	3.15	13.48	6.20	8. 3	9.40	5.26	15.30	3.23

App. Alt.	Ref.	App. Alt.	Ref.	App. Alt.	Ref.	App. Alt.	Ref.
D. M.	M. S.	D.	M. S.	D.	M. S.	D.	M. S.
16. 0	3.17	30	1.38	50	0.48	70	0.21
16.30	3.11	31	1.35	51	0.46	71	0.20
17. 0	3. 5	32	1.31	52	0.45	72	0.19
17.30	2.59	33	1.28	53	0.43	73	0.17
18. 0	2.54	34	1.24	54	0.41	74	0.16
18.30	2.49	35	1.21	55	0.40	75	0.15
19. 0	2.44	36	1.18	56	0.38	76	0.14
19.30	2.40	37	1.16	57	0.37	77	0.13
20. 0	2.36	38	1.13	58	0.36	78	0.12
20.30	2.32	39	1.10	59	0.34	79	0.11
21. 0	2.28	40	1. 8	60	0.33	80	0.10
21.30	2.24	41	1. 5	61	0.32	81	0. 9
22. 0	2.20	42	1. 3	62	0.30	82	0. 8
23. 0	2.14	43	1. 1	63	0.29	83	0. 7
24. 0	2. 7	44	0.59	64	0.28	84	0. 6
25. 0	2. 2	45	0.57	65	0.27	85	0. 5
26. 0	1.56	46	0.55	66	0.25	86	0. 4
27. 0	1.51	47	0.53	67	0.24	87	0. 3
28. 0	1.47	48	0.51	68	0.23	88	0. 2
29. 0	1.43	49	0.50	69	0.22	89	0. 1

Extensive factor for finding the circumference of a circle, when the diameter is given, by multiplying it into the factor.

3.14159265358979323846264338327950288419716939937510
582097494459230781640628620899862803482534211706798214
80865132723066470938446+

Long Measure.

7.92 inches make one link.
100 links (66 feet) one chain.
80 chains make one mile.

Superficial Measure.

10 square chains make one acre.
640 square acres make one square mile.

Cubic Measure computed in Weight.

28.8 cubic inches (one pint) of pure water weigh one pound.

A cubic foot of pure water weighs 60 pounds.

2000 pounds (2000 pints) make one ton.

Note. This is according to the revised statutes of New-York. In England, and many of the States, a cubic foot of water weighs 62.5 ℔. A ton, 2240 ℔.

Five gallons per day is allowed in Paris (France) for each individual, at average, by the waterworks companies. In Troy $3\frac{1}{2}$ gallons per day. Partly supplied by cisterns and wells.

Cord Wood.

Multiply length, breadth, and thickness together, and allow 128 cubic feet for a cord. If the measure is in feet and inches, multiply the length 8 f. 4 in. breadth 4 f. 7 in. height 3 f. 9 in. This is called duo-decimal rule.

8 4	38 2 4	128)143 2 9(1
4 7	3 9	128
33 4	114 7 0	15 2 9
4 10 4	28 7 9 0	
38 2 4	143 2 9 0	1 cord, 15 f. 3 in.

Bushel Measures.

A bushel of charcoal, if birch, weighs 45.75 ℔.—if oak, 28 ℔. To find the number of bushels in a load of charcoal, find the contents in cubic inches, as directed in sections 221 and 222. Divide the contents by 2339 (the cubic inches in a bushel) the quotient will be the measure in bushels. But an allowance of 14 per cent. for shaking on a wagon is allowed, if it has been moved a considerable distance. 114 bushels, on being drawn from Sandlake, was shaken down to 100 bushels. Apples will shake down about 4 per cent.—also ears of corn and potatoes.

Charcoal ought always to be sold by weight. Let one bushel be weighed for a standard; then weigh the load on the hay-scales; then re-weigh after the quantity sold is taken out.

Comparing Rail-Road Curves, from sec. 217.

When radius 2000 feet, arc 500 feet.
When radius 1500 feet, arc 666.66 feet.
When radius 800 feet, arc 1250 feet.
All describe areas of 500.000 feet each.

```
        2000 rad.
         500 arc.
        ---------
      2)1,000,000 double area.
        ---------
         500.000 area.

rad. 1500)1,000,000 double area.
          ---------
           666.66 arc with radius of 1500 feet.

rad. 8.00)1,000,000 double area.
          ---------
           1250 arc with radius of 800 feet.
```

W. G. L.

From section 287.

Formula for pipes when the diameter is sought.

The quantity of water discharged per second is 3 cubic feet.
The length of the pipe 4000 feet.
Head, or descent 10 feet.

```
3 cubic feet.          38.116
3                      38.116
-                     -------
9                     228696
4000                  38116
-----                38116
36000              304928
                  114348
                  -------
```

$38.116^2 =$ $14528.29456 \times 10 = 14528.294560$

```
14528.29)36000.000000(2.47792
         2905658
         -------
          6943420
          5811316
          --------
          11321040
          10169803
          --------
           11512370
           10159803
           --------
            13425670
            13075461
            --------
              3502090
              2905658
```

Extracting the root according to Hutton's method for the higher powers. See sec. 30.

Assuming 1.19 for the root.

1.19 is raised to the fifth power 2.386.

2.386		2.47792				
2		2				
4.772		4.95584				
2.47792		2.386				
7.24992	:	7.34186	: :	1.19	:	1.2

Second statement :

1.2^5=2.48832	:	2.47792				
2		2				
4.97664		4.95584				
2.47792		2.48832				
5.45456	:	7.44416	: :	1.2	:	1.1983.

J. O.

Calculation, from section 288.

Formula for open canals when the velocity and quantity are required.

Head 10 feet.
Length 30 feet.
Area 4.8.
Perimeter 6.8.

$10 \times 4.8 = 48.$
$30 \times 6.8 = 204.)48.00009(.23529$
9582

47058
188232
117645
211761

2254.54878
.0111

2254.55988(47.482		
16	0.109	*
87)654	47.373	velocity in feet per sec.
609	4.8	transverse area.
944)4555	378984	
3776	189492	
9488)77998	227.3904	cubic feet × 60 ℔. =
75884		13643.4240 ℔. per sec.
94862)211480		E. N. H.
189724		

* In section 288, this formula is *wrongly* directed to be added.

Calculation of the height of the Atmosphere.

From sections 292 and the sixth subdivision.

Hours.		Min.	
24	:	360° ::	70
60		70	
Min. 1440		1440)25200(17°.5=17° 30′	

```
1440)25200(17°.5=17° 30′
     1440
     -----
     10800
     10080
     -----
      7200
      7200
```

Sine of 81° 15′		Sem. Diam. Miles.		Sine of 90°
.98836	:	4000	::	1.00000
				4000

```
                                  Miles.
.98836)4000.00000(4047.1   from earth's
       395344              centre to the top of
       ------              atmosphere.
        465600
        395344
        ------
         702560
         691852
         ------
          107080
           98836
```

4047.1
4000 earth's radius.

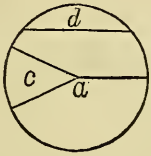
47.1 height of atmosphere. Generally estimated at 45 miles.

A. B. C.

WOOD-CUT FIGURES;

Explaining and familiarizing some of the subjects treated of in the preceding sections.

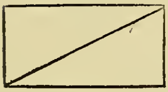
Lines are right or curved. A *right line* is the shortest measure between two points; as in the figure. A *curved line* is always a line, which, if sufficiently continued, will return into itself, and form a circle; as each of the curves in the figure.



Circle, is a figure bounded by a continued line, every where equi-distant from a point in the centre. *c* sector, bounded by two radii and an arc; *d* segment, bounded by a chord line and an arc; *a* the centre.

Ellipse, made by moving one pin's point around two focal ones; and is kept in the periphery by sliding around in the loop of a thread.

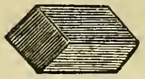
Square, is a figure of four equal sides meeting at right angles.



Rectangle, (or parallelogram) a long square with opposite sides only equal. The line connecting opposite corners is a *diagonal.*

Rhomb, a figure with four opposite equal sides, not meeting at right angles.

Superficies, having length and breadth without thickness.



Solid, having length, breadth, and thickness.

Right angle, a square corner. Sec. 31.

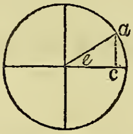
Obtuse angle, opens wider than a right angle. Sec. 31.

Acute angle, does not open as wide as a square. Sec. 31.

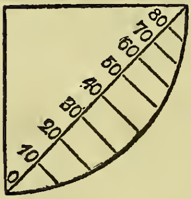
A right-angled triangle contains one right angle. Sec. 32.

Obtuse-angled triangle has one obtuse angle. Sec. 32.

Acute-angled triangle has all three angles acute. If it has two *equal sides,* it should be called also *Isosceles* triangle. If the three sides are equal, it is also called *Equilateral.* Sec. 32.



a c is the sine of the angle *e*. Sec. 33, article 3.



The figured line is a line of chords. Sec. 33, article 4.

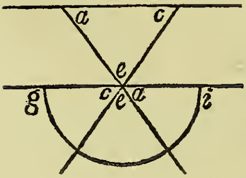
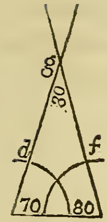
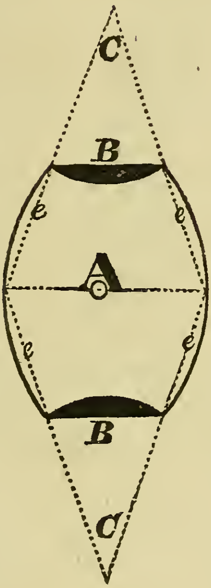


Illustration of the principle that a triangle contains 180 degrees. Semi-circle *g i* is the measure of 180°—angle *a* in it equals angle *a* in the triangle—*e* equals *e*—*c* equals *c*. Sec. 33, article 5.

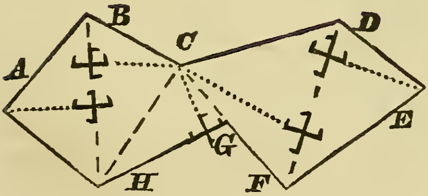


Geometrical trigonometry, is illustrated by plotting this figure; having the side given, which is drawn from the angle at 80 to the angle at 70, and drawing the lines up to *g*, through the marked points on the arcs *d* and *f*, until they cross at *g*. See sec. 34.



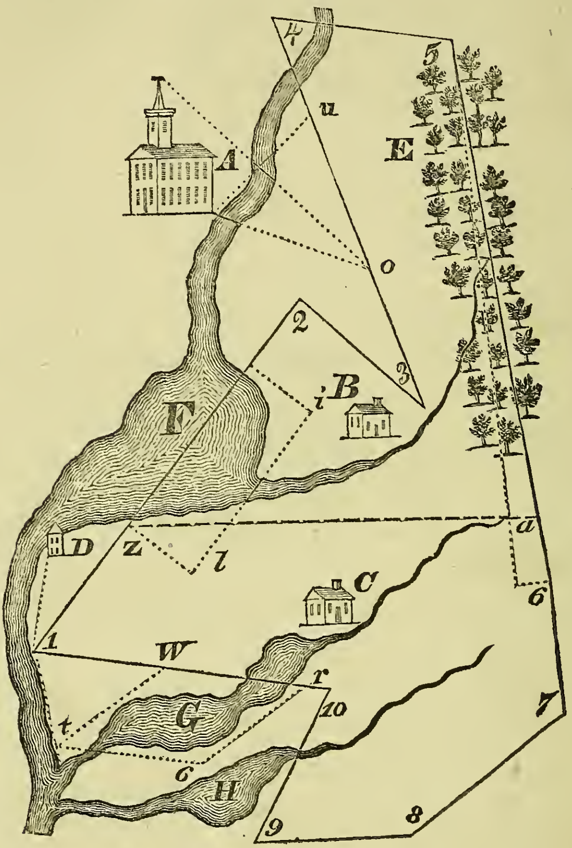
Frustrum of a cone or pyramid, found by calculating the pyramid as if topped out; and then calculating and subtracting the added point. See sec. 53, article 5.

This figure also illustrates the 7th article, Guaging, under the 53d section.

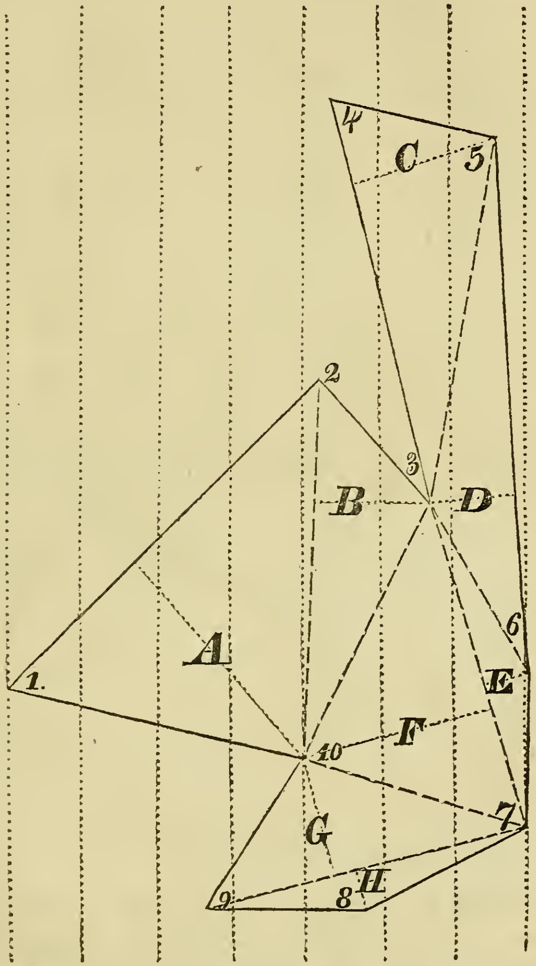


Field surveying with a cross, where a farmer is desirous to know the contents of a field for planting seed, to pay for mowing by the acre, &c. See sec. 60 and 61.

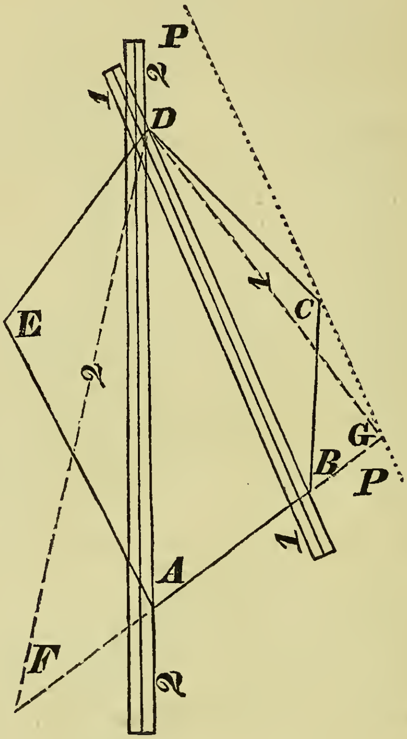
Map of a farm referred to from sections 74, 75, 78, and then in all the sections to sec. 89. And again from section 106 to 109, where *heights* and *distances* and *division of land* are explained.



Plot of a survey referred to from sections 91 to 95 ; wherein the method of reducing and raising the scale, plotting, *triangular cuttings and castings* are explained.



Plot of a survey, wherein the area is cast up by reducing the whole survey of *a single triangle*. This is referred to and explained from sections 96 to 99.

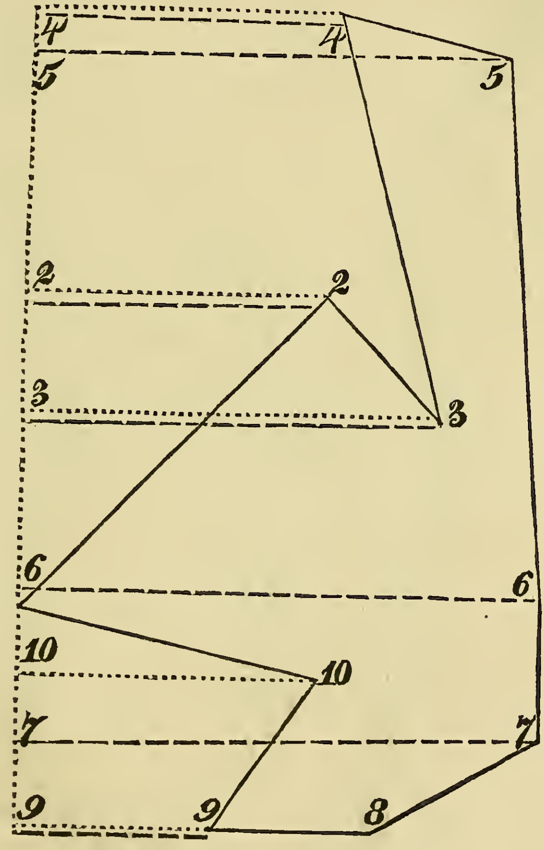


Contraction of the vein, from sec. 315.

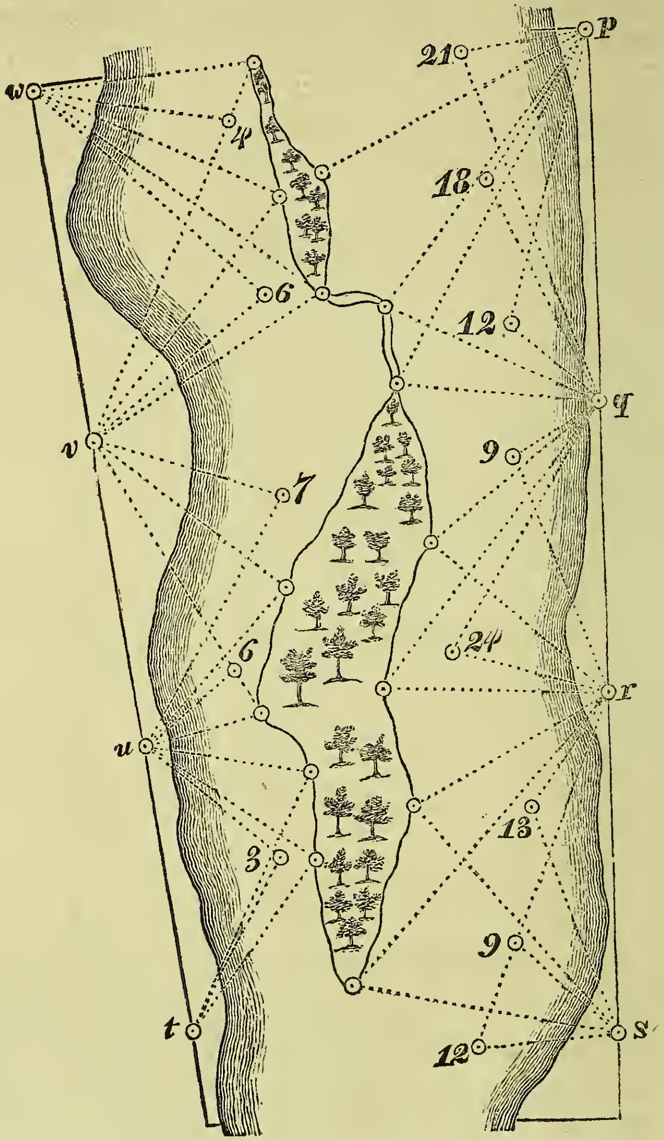
One third of the area is generally deducted for the contraction of the vein. This expressed decimally is 0.666—this agrees nearly with Bossut's experiments. Eytelwein adopts 0.640. It has been demonstrated by experiment, that a conical tube, whose length is about 0.88, the diameter of its base, if adjusted to the aperture, will reduce the contraction of the vein to about one sixth—that is, the

area of the contracted vein will be but one sixth less than that of the area of the aperture. Also, that a gate-hole through a four-inch plank, cut a little convergingly, will add much to the efflux of a column of water; by lessening the contraction of the vein.

Plot of a survey calculated by the *trapezoidal method*—referred to from sections 100 to 104.



Harbor survey; being a section of Hudson river, above Troy. Referred to from sections 120 to 126.



For sec. 198 to 200, on Rail-Road Curves.

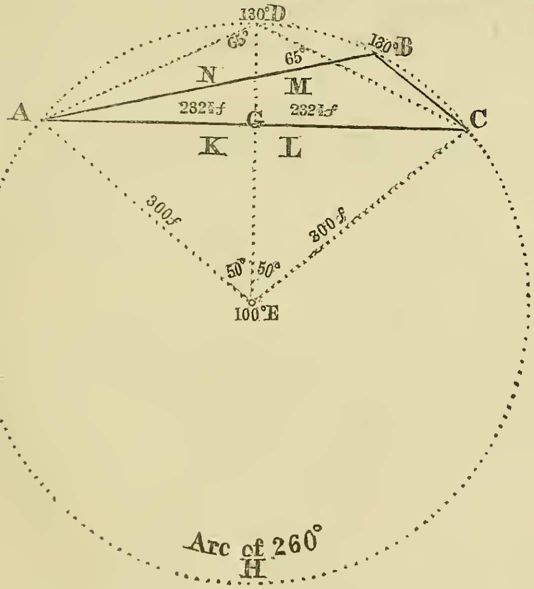
Two propositions, referred to in sec. 199, *are here given at full length.*

1. Two chord lines meeting in the periphery of a circle, form an angle, which is measured by an arc, half as long as the arc required to connect the ends of two radii, which meet the ends of the chord lines. As the arc A H C is twice as long as an arc required to measure the angle A D C.

2. Two sub-chord lines meeting at any point in the periphery of a circle, the point of meeting will continue to form the same angle, if moved to any other point of the periphery, on the same side of the general chord line. As the angle 130° at B, is 130° at D.

Description of the figure referred to sec. 198 *to* 200.

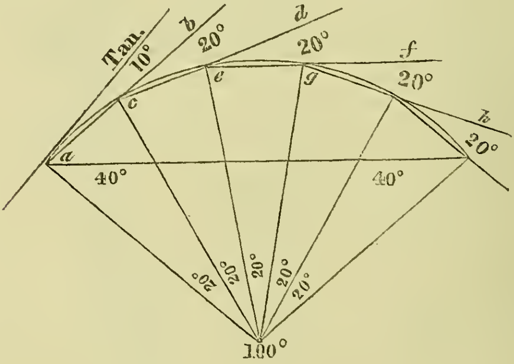
Scale 200 feet per inch. Radius E C 300 feet. General chord line A C 465 feet. Sub-chord line A B is a traverse line 360 feet long, taken in the field. Sub-chord line B C is a traverse line 132



feet long, taken in the field. Angle B at the meeting of the traverse lines, is 130°; as found by considering the compass directions of each. Moved to D, it continues to be 130°, according to the second proposition above. This angle doubled makes 260°, and is measured by the arc A H C, according to the first proposition above. This subtracted from 360°, gives the angle 100° at E. The diagonal line E D halves the angles at E and D. The general chord line A C, being halved by said diagonal line, give horizontal legs to four right-angled triangles; to wit, K L M N, each being 232½ feet.

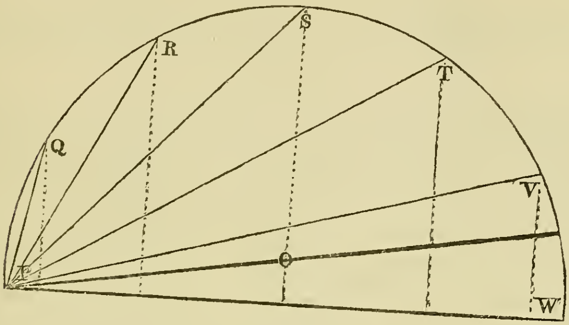
Above description extended to the second figure; which is referred to sec. 201 *to* 204.

Tan. the tangent line, from which the line *a b* is deflexed 10°, a number equal to half the angle (20°) of the isosceles triangle at the centre of the circle. The next line *c d* is deflexed from the chord line *a c* 20°, a number equal to the angle at the apex of the isosceles triangle at the centre of the circle. All the remaining deflexions are 20° also; as they are deflexed from the last preceding chord line, which is double the deflexion from the tangent line.



Rail-Road Curve. Illustration of sec. 207.

The dotted lines are used in sec. 208. This example comprises more than half a circle; of course the line P W is shorter than P V. In practice no curve ever includes half a semi-circle.



Sliding Rule.

The sliding rule has four lines—two stationary on the wooden part, two sliding ones on the brass slip. The upper one on the wood is marked A—the upper one on the brass is marked B—the lower one on the brass is marked C—the lower one on the wood is marked D, and called girt line.

Measuring boards.

1. Take the width of the board in inches.
2. Find the number agreeing with the number of inches on line A, and slide figure 12 to it on B.
3. Read off the square feet by measuring the length of the board in feet and inches, and finding the number agreeing with it on line B, and against it on A, read the square measure of the board in feet.

Measuring timber, square or round.

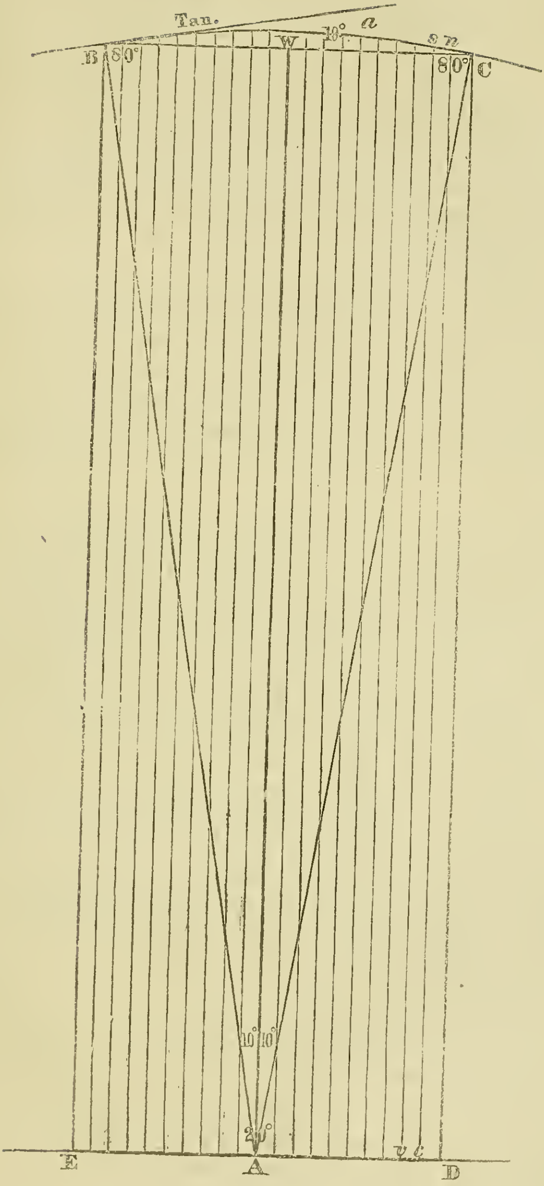
1. Take the length of the timber in feet and inches.

2. Find the number agreeing with the number of feet and inches on the line C, and slide it to the figure 12 on D.

3. Read off the cubic feet by finding a quarter of the girt in inches and finding a number agreeing with the inches of the quarter girt on the line D, and against it on C, read the cubic contents of the timber in feet.

For sections from 211 *to* 214. *Illustration of the Calculation of Ordinates.*

B C the base (100 feet) A B and A C the two equal sides (300 feet each) of an isosceles triangle. A, the angle at the apex of the isosceles triangle (20°) being double the angle formed by the deflexion of the chord line B C from the tangent line B *a* (10°.) The angle at A is at the centre of the circle, of which B 10 C is an arc. E D is a middle portion of the horizontal diameter of the circle; which, in its whole length, is double the radii A B and A C = 600 feet. E D being equal to the chord line of the given arc (100 feet) it has 50 feet on each side of the centre A. Therefore the distance from D to the end of the diameter in the direction of E, is 350 feet, and the remainder in the opposite direction is 250 feet. By a known principle in mathematics, if 350 is multiplied by 250, and the square root of the product extracted, it will give the length of the ordinate D C. In the same manner the line E B is found. Shorten the side in the direction of E, 5 feet to *c*, leaving that line 345 feet, and making the other 255, and intermultiply them and extract the square root as before, you obtain the ordinate *c n*. Subtract the standing ordinate D C from the ordinate *c n*, and the remainder will be that part of the ordinate which is above the chord line B C. Proceed in the same manner with *v s*, and all other measures on the diameter line E D, moving 5 feet at a time along said line. In this manner all the offset lines (called ordinates) above the said chord line, are obtained, as far as the middle ordinate A W. Then by inverting their order, all between W and B may be set down. In this manner the table under section 210 was made.


Tan.
W
10°
a
s n
B
80°
80°
C
10°
10°
20°
v t
E
A
D

For sections 224, 225 and 226.

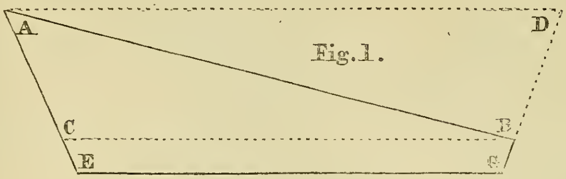
In taking the *cross areas of excavations and embankments*, it is generally preferable, when very irregular, to suppose the base and surface level, and of course, parallel, however uneven they may be in reality. Then add their *calculated* lengths, halve their sum, and multiply that by the distance between their levels. This *imaginary* trapezoidal result is then to be reduced to the truth, by casting and subtracting the vacant places.

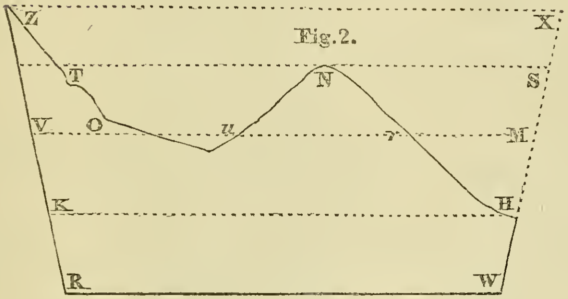
Example, Fig 1. A B is a side hill. Cast the trapezoid B C E G in the usual way. Cast the triangle A B C in the usual way, when a triangle is made between two parallel lines, A D and B C. Or the whole trapezoid, A D E G, may be cast, and the triangle A D B be deducted. ☞ In all cases the *distance* between parallel lines, and between an apex and a base, the difference of levels is the distance.

Example, Fig. 2. Z T O *u* N H is the uneven surface of a piece of required excavation. Add Z X and R W, halve their sum, and multiply that by the distance between the ascertained level of the base R W, and the level of the highest point to be excavated, Z X; then cast and subtract the vacant spaces. First, cast the trapezoid Z X T S, in the usual way. Second, cast triangle H N S, whose base is calculated from the central *bench* (fixed stake) and whose perpendicular is the difference in level between the apex at H and the base N. Third, consider the figure T N O *u*, as a trapezoid, and calculate it as such; for though the side O *u* is not parallel to T N, it is a case which may be averaged by the levelled line O M being made to be intermediate in height between the true levels of O and *u*. Fourth, add the areas of these two trapezoids and the triangle, and subtract their sum from the factitious area of the whole assumed trapezoid.

Remark. This transverse area may also be cast, *directly*, by casting the trapezoid R W K H—then the trapezoid K H V *r*—then the triangle V O Z—then, last, make an average triangle *u r* N. But the point would require a special measure, and both ends of the base line *u r*, would require more time and become more complicated, than by adopting the deduction method. In all cases the sur-

face is most accessible; consequently may generally be easiest measured. Besides, the engineer can always construct his general ideal trapezoid in perfection, and fix some points of it to his *benches.*





Power to overcome friction in a flouring mill, from sec. 315, *continued.*

Suspend weights on a water-wheel at its periphery on a horizontal level with its axis, cog-wheel, or other vertical wheel, until the whole gearing, stone, &c., start. Then calculate the wheel and axil power, so as to compare the advantage at the point of the application of the weight, with the point (or average point) where the water acts on the wheel, whether overshot, undershot, or horizontal.

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